Abstract—An adaptive variable structure position control for induction motors using field oriented control theory is presented. The proposed sliding-mode control law incorporates an adaptive switching gain to avoid calculating an upper limit of the system uncertainties. The design incorporates a flux estimator that operates on the principle of flux and current observer. The proposed observer is basically an estimator that uses a plant model and a feedback loop with measured stator voltages and currents. The stability analysis of the proposed controller under parameter uncertainties and load disturbances is provided using the Lyapunov stability theory. Finally, experimental results show that the proposed controller with the proposed observer provides high-performance dynamic characteristics and that this scheme is robust with respect to plant parameter variations and external load disturbances.

I. INTRODUCTION

Field oriented control method is widely used for advanced control of induction motor drives [1]. The field-oriented technique guarantees the decoupling of torque and flux control commands of the induction motor, so that the induction motor can be controlled linearly as a separated excited D.C. motor. However, the control performance of the resulting linear system is still influenced by uncertainties, which usually are composed of unpredictable parameter variations, external load disturbances, and unmodelled and nonlinear dynamics. Therefore, many studies have been made on the motor drives in order to preserve the performance under these parameter variations and external load disturbance, such as nonlinear control, optimal control, predictive control, variable structure system control, adaptive control, fuzzy control and neural control [2]-[6].

The sliding-mode control strategy has been focussed on many studies and research for the position control of the induction motors [7]-[9]. However, the traditional sliding control schemes requires the prior knowledge of an upper bound for the system uncertainties since this bound is employed in the switching gain calculation. This upper bound should be determined as precisely as possible, because as higher is the upper bound higher value should be considered for the sliding gain, and therefore the control effort will also be high, which is undesirable in a practice. In order to surmount this drawback, in the present paper it is proposed an adaptive law to calculate the sliding gain which avoids the necessity of calculate an upper bound of the system uncertainties.

This manuscript is organized as follows. The flux observer is introduced in Section 2. Then, the proposed adaptive variable structure robust position control is presented in Section 3. In the Section 4, some simulation results are presented. Finally, concluding remarks are stated in Section 5.

II. ROTOR FLUX ESTIMATOR

Many schemes [10] based on simplified motor models have been devised to estimate some internal variables of the induction motor from measured terminal quantities. This procedure is frequently used in order to avoid the presence of some sensors in the control scheme. In order to obtain an accurate dynamic representation of the motor, it is necessary to base the calculation on the coupled circuit equations of the motor.

Since the motor voltages and currents are measured in a stationary frame of reference, it is also convenient to express the induction motor dynamical equations in this stationary frame.

The system state space equations can be written in the form [11]:

$$\dot{x} = Ax + Bv_s$$

where

$$x = [i_{ds}, i_{qs}, \psi_{dr}, \psi_{qr}]^T$$

$$v_s = [v_{ds}, v_{qs}, 0, 0]^T$$

$$B = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & 0 \end{bmatrix}^T$$

$$A = \begin{bmatrix} -\rho & 0 & R_T & \frac{w_r}{c} \\ 0 & -\rho & -\frac{w_r}{c} & R_T \\ \frac{L_m R_T}{L_r} & 0 & -\frac{R_T}{L_r} & -w_r \\ 0 & \frac{L_m R_T}{L_r} & w_r & -\frac{R_T}{L_r} \end{bmatrix}$$
where \( \rho = \frac{L_m^2 R_c + L_e^2 R_s}{\sigma L_s L_r^2} \) and \( c = \frac{\sigma L_s L_r}{L_m} \).

Considering the stator currents as the system output, the output equation for this system is:

\[
y = C \cdot \hat{x}
\]

where

\[
C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\]

Then, the states observer, which estimates the system states (stator current and rotor flux), is defined by means of the following equation (Luenberger observer):

\[
\dot{\hat{x}} = A \hat{x} + B v_s + G (y - C \hat{x})
\]

\[
= A \hat{x} + B v_s + G C (x - \hat{x})
\]

where the symbol \( \hat{\cdot} \) represents the estimated values and \( G \) is the observer gain matrix.

Therefore, if the observer gain \( G \) is adequately chosen, then the estimation error converges to zero. Consequently the estimated states \( \hat{i}_{ds}, \hat{i}_{qs}, \hat{\psi}_{ds}, \hat{\psi}_{qs} \) converges to the real states \( i_{ds}, i_{qs}, \psi_{ds}, \psi_{qs} \) as \( t \) tends to infinity. Hence, the rotor flux may be obtained from the state observer given by equation (3).

III. ADAPTIVE VARIABLE STRUCTURE POSITION CONTROL

The mechanical equation of an induction motor can be written as:

\[
J \ddot{\theta}_m + B \dot{\theta}_m + T_L = T_e
\]

where \( J \) and \( B \) are the inertia constant and the viscous friction coefficient of the induction motor respectively; \( T_L \) is the external load; \( \theta_m \) is the rotor mechanical position, which is related to the rotor electrical position, \( \theta_r \), by \( \theta_m = 2 \theta_r / p \) where \( p \) is the pole numbers and \( T_e \) denotes the generated torque of an induction motor, defined as [10]:

\[
T_e = \frac{3p L_m}{4 L_r} (\dot{\psi}_{dr} \dot{i}_{qs} - \dot{\psi}_{qr} \dot{i}_{ds})
\]

where \( \dot{\psi}_{dr} \) and \( \dot{\psi}_{qr} \) are the rotor-flux linkages, with the subscript ‘e’ denoting that the quantity is referred to the synchronously rotating reference frame; \( \dot{i}_{ds} \) and \( \dot{i}_{qs} \) are the stator currents, and \( p \) is the pole numbers.

The relation between the synchronously rotating reference frame and the stationary reference frame is performed by the so-called reverse Park’s transformation:

\[
\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \begin{bmatrix} \cos(\theta_e) & -\sin(\theta_e) \\ \cos(\theta_e - 2\pi/3) & -\sin(\theta_e - 2\pi/3) \\ \cos(\theta_e + 2\pi/3) & -\sin(\theta_e + 2\pi/3) \end{bmatrix} \begin{bmatrix} x_d \\ x_q \end{bmatrix}
\]

where \( \theta_e \) is the angle position between the d-axis of the synchronously rotating reference frame and the a-axis of the stationary reference frame, and it is assumed that the quantities are balanced.

The estimated angular position of the rotor flux vector (\( \hat{\psi}_r \)) related to the d-axis of the stationary reference frame may be calculated by means of the rotor flux components in this reference frame (\( \psi_{dr}, \psi_{qr} \)) as follows:

\[
\hat{\theta}_e = \arctan \left( \frac{\psi_{qr}}{\psi_{dr}} \right)
\]

where \( \hat{\theta}_e \) is the estimated angular position of the rotor flux vector.

Using the field-orientation control principle [10], the current component \( i_{ds} \) is aligned in the direction of the rotor flux vector \( \psi_r \), and the current component \( i_{qs} \) is aligned in the perpendicular direction to it, then:

\[
\psi_{dr} = 0, \quad \psi_{ds} = |\psi_r|
\]

Taking into account the results presented in equation (9), the induction motor torque eqn.(6) is simplified to:

\[
T_e = \frac{3p L_m}{4 L_r} \psi_{dr} \psi_{ds}
\]

where \( K_T \) is the torque constant, defined as follows:

\[
K_T = \frac{3p L_m}{4 L_r} \psi_{dr}
\]

where \( \psi_{dr} \) denotes the command rotor flux.

With the above mentioned proper field orientation, the dynamics of the rotor flux is given by [10]:

\[
\frac{d\psi_{dr}}{dt} + \frac{\psi_{dr}}{T_r} = \frac{L_m}{T_r} \dot{i}_{ds}
\]

where the parameters are defined as:

\[
a = \frac{B}{J}, \quad b = \frac{K_T}{J}, \quad f = \frac{T_L}{J};
\]

Now, we are going to consider the previous mechanical equation (13) with uncertainties as follows:

\[
\ddot{\theta}_m = -(a + \Delta a) \dot{\theta}_m - (f + \Delta f) + (b + \Delta b) i_{qs}
\]

where the terms \( \Delta a, \Delta b \) and \( \Delta f \) represents the uncertainties of the terms \( a, b \) and \( f \) respectively.

Let us define the position tracking error as follows:

\[
e(t) = \theta_m(t) - \theta^*_m(t)
\]

where \( \theta^*_m \) is the rotor position command.

Taking the second derivative of the previous equation with respect to time yields:

\[
\ddot{e}(t) = \ddot{\theta}_m - \ddot{\theta}^*_m = u(t) + d(t)
\]

where the signal \( u(t) \) collets the known terms,

\[
u(t) = b \dot{i}_{qs}(t) - a \dot{\theta}_m(t) - f(t) - \ddot{\theta}^*_m(t)
\]

and the signal \( d(t) \) collets the uncertainty terms,

\[
d(t) = -\Delta a w_m(t) - \Delta f(t) + \Delta b \dot{i}_{qs}(t)
\]
Now, we are going to define the sliding variable \( S(t) \) as:

\[
S(t) = \dot{e}(t) + k_1 e(t) + k_i \int e(t) \, dt
\]  

(20)

where \( k \) and \( k_i \) are positive constant gains.

The variable structure position controller is designed as:

\[
u(t) = -k \dot{e}(t) - k_i e(t) - \dot{\beta} \gamma \text{sgn}(S)
\]  

(21)

where the \( k \) is the previously defined gain, \( \dot{\beta} \) is the switching gain, \( S \) is the sliding variable defined in eqn. (20) and \( \text{sgn}(\cdot) \) is the sign function.

Finally, the switching gain \( \dot{\beta} \) is adapted according to the following updating law:

\[
\dot{\beta} = \gamma |S| \quad \dot{\beta}(0) = 0
\]  

(22)

where \( \gamma \) is a positive constant that let us choose the adaptation speed for the sliding gain.

In order to obtain the position trajectory tracking, the following assumption should be formulated:

\[(A1) \quad \text{There exits an unknown finite and positive switching gain } \beta \text{ such that } \beta > d_{\text{max}} + \eta, \text{ where } d_{\text{max}} \geq |d(t)| \quad \forall t \text{ and } \eta \text{ is a positive constant.}\]

Note that this condition only implies that the system uncertainties are bounded magnitudes.

**Theorem 1:** Consider the induction motor given by equation (15). Then, if the assumption \((A1)\) are verified, the control law (21) leads the rotor mechanical position \( \theta_m(t) \) so that the position tracking error \( e(t) = \theta_m(t) - \theta^*_m(t) \) tends to zero as the time tends to infinity.

The proof of this theorem will be carried out using the Lyapunov stability theory.

**Proof:** Define the Lyapunov function candidate:

\[
V(t) = \frac{1}{2} \dot{S}(t)S(t) + \frac{1}{2} \dot{\beta} \dot{S}(t)
\]  

(23)

where \( S(t) \) is the sliding variable defined previously and \( \dot{\beta}(t) = \dot{\beta} - \beta \).

Its time derivative is calculated as:

\[
\dot{V}(t) = S(t) \dot{S}(t) + \dot{\beta}(t) \dot{S}(t)
\]

\[=
S \cdot [\dot{e} + k \dot{e} + k_i e] + \dot{\beta}(t) \dot{S}(t)
\]

\[=
S \cdot [u + d + k \dot{e} + k_i e] + \dot{\beta} \gamma |S|
\]

\[=
S \cdot [-k \dot{e} - k_i e - \dot{\beta} \gamma \text{sgn}(S) + d + k \dot{e} + k_i e]
\]

\[+ (\dot{\beta} - \beta) \gamma |S|
\]

\[= dS - \dot{\beta} \gamma |S| + \dot{\beta} \gamma |S| - \beta \gamma |S|
\]

\[\leq |d||S| - \beta \gamma |S|
\]

\[\leq |d| |S| - (d_{\text{max}} + \eta) \gamma |S|
\]

\[= |d||S| - d_{\text{max}} \gamma |S| - \eta \gamma |S|
\]

\[\leq -\eta \gamma |S|
\]  

(24)

then

\[
\dot{V}(t) \leq 0
\]  

(26)

It should be noted that the eqns. (17), (20), (21) and (22), and the assumption \((A1)\) have been used in the proof.

Using the Lyapunov’s direct method, since \( V(t) \) is clearly positive-definite, \( \dot{V}(t) \) is negative semidefinite and \( V(t) \) tends to infinity as \( S(t) \) and \( \dot{\beta}(t) \) tends to infinity, then the equilibrium at the origin \([S(t), \dot{\beta}(t)] = [0, 0]\) is globally stable, and therefore the variables \( S(t) \) and \( \dot{\beta}(t) \) are bounded. Since \( S(t) \) is bounded then it is deduced that \( e(t) \) and \( \dot{e}(t) \) are bounded.

On the other hand, making the derivative of equation (20) it is obtained,

\[
\dot{S}(t) = \ddot{e}(t) + k \dot{e}(t) + k_i e(t)
\]  

(27)

then, substituting the equation (17) and (21) in the above equation,

\[
\dot{S}(t) = u(t) + d(t) + k \dot{e}(t) + k_i e(t)
\]

\[= -k \dot{e} - k_i e - \dot{\beta} \gamma \text{sgn}(S) + d(t) + k \dot{e} + k_i e
\]

\[= d(t) - \beta \gamma \text{sgn}(S)
\]  

(28)

From equation (28) we can conclude that \( \dot{S}(t) \) is bounded because \( d(t), \gamma \) and \( \beta \) are bounded.

Now, from equation (24) it is deduced that

\[
\dot{V}(t) = d \dot{S}(t) - \beta \gamma \frac{d}{dt} |S(t)|
\]  

(29)

which is a bounded quantity because \( \dot{S}(t) \) is bounded.

Under these conditions, since \( \dot{V}(t) = \dot{V}(t) \) is a uniformly continuous function, so Barbalat’s lemma let us conclude that \( \dot{V} \rightarrow 0 \) as \( t \rightarrow \infty \), which implies that \( S(t) \rightarrow 0 \) as \( t \rightarrow \infty \).

From When \( S(t) = \dot{S}(t) = 0 \), the dynamic behavior of the tracking problem is equivalently governed by the following equation obtained from (27):

\[
\dot{S}(t) = \ddot{e}(t) + k \dot{e}(t) + k_i e(t) = 0
\]  

(30)

Then, like \( k \) and \( k_i \) are positive constants, the tracking error \( e(t) \) and its derivative \( \dot{e}(t) \) converges to zero exponentially.

It should be noted that, a typical motion under sliding mode control consists of a *reaching phase* during which trajectories starting off the sliding surface \( S = 0 \) move toward it and reach it, followed by *sliding phase* during which the motion will be confined to this surface and the system tracking error will be represented by the reduced-order model (30), where the tracking error tends to zero.

Finally, the torque current command, \( i_{qp}^*(t) \), can be obtained directly substituting eqn. (21) in eqn. (18):

\[
i_{qp}^*(t) = \frac{1}{\beta} \left[-k \dot{e} - k_i e - \dot{\beta} \gamma \text{sgn}(S) + \dot{\theta}_m + \dot{\theta}_m^* + f(t)\right]
\]  

(31)

Therefore, the proposed variable structure control resolves the position tracking problem for the induction motor in presence of some uncertainties in mechanical parameters and load torque.
IV. SIMULATION RESULTS

In this section we will study the position regulation performance of the proposed sliding-mode field oriented control versus reference and load torque variations by means of simulation examples. The block diagram of the proposed robust position control scheme is presented in Figure 1 and the function of the blocks that appear in this figure are explained below: The block ‘VSC Controller’ represents the proposed sliding-mode controller, and it is implemented by equations (20) and (31). The block ‘limiter’ limits the current applied to the motor windings so that it remains within the limit value, being implemented by a saturation function. The block ‘\(dq\rightarrow abc\)’ makes the conversion between the synchronously rotating and stationary reference frames (Park’s Transformation). The block ‘Current Controller’ consists of a SVPWM current control. The block ‘SVPWM Inverter’ is a six IGBT-diode bridge inverter with 540 V DC voltage source. The block ‘Field Weakening’ gives the flux command based on rotor speed, so that the PWM controller does not saturate. The block ‘\(\hat{\psi}_{d}\) Calculation’ provides the current reference \(\hat{\psi}_{d}\) from the rotor flux reference through the equation (12). The block ‘Flux Estimator’ represents the proposed flux estimator, and it is implemented by the equation (3). The block ‘\(\hat{\theta}\) Calculation’ provides the angular position of the rotor flux vector. Finally, the block ‘IM’ represents the induction motor.

The induction motor used in this case study is a 50 HP, 460 V, four pole, 60 Hz motor having the following parameters: \(R_s = 0.087 \Omega\), \(R_r = 0.228 \Omega\), \(L_s = 35.5 mH\), \(L_r = 35.5 mH\), and \(L_m = 34.7 mH\).

The system has the following mechanical parameters: \(J = 1.662 kg.m^2\) and \(B = 0.1 N.m.s\). It is assumed that there are an uncertainty around 20 % in the system parameters, that will be overcome by the proposed sliding control.

In addition the following values have been chosen for the controller parameters: \(k = 50\), \(k_i = 30\), \(\gamma = 30\) and \(\hat{\beta}(0) = 0\).

In this example the motor starts from a standstill state and we want that the rotor position follows a ramp command, that starts from 0 rad and finish at 2 rad. The system starts with an initial load torque \(T_L = 50 N.m\), and at time \(t = 1.5 s\), the load torque steps from \(T_L = 50 N.m\) to \(T_L = 250 N.m\), and as before, it is assumed that there is an uncertainty around 20 % in the load torque.

![Fig. 1. Block diagram of the proposed sliding-mode field oriented control](image1)

![Fig. 2. Reference and real rotor position signals (rad)](image2)
is too small for the new uncertainty introduced in the system due to the load torque increment. But, after a small time, the sliding gain is adapted so that this gain can compensate for the new system uncertainties and then the rotor position error is eliminated.

Figure 4 shows the rotor speed. This figure presents an increasing initial speed value but after $t = 0.2\,\text{s}$ the value decreases to zero at $t = 0.7\,\text{s}$ because the rotor position reach the reference value.

Figure 5 shows the motor torque. The motor torque has a high initial value in the speed acceleration zone because it is necessary a high torque to increment the rotor speed and then the value decreases in a deceleration region. Later at time $t = 1.5\,\text{s}$ the torque increases due to the load torque increment.

This figure shows that the so-called chattering phenomenon appears in the motor torque. Although this high frequency changes in the torque will be reduced by the mechanical system inertia, they could cause undesirable vibrations in the rotor, which may be a problem for certain systems. However, for the systems that do not support this chattering, it may be eliminated substituting the sign function by the saturation function in the control signal.

Figure 6 shows the stator current $i_{sa}$. As in the case of the motor torque, the current signal presents a high value in the initial state. Next, in the constant position region the current is lower because the motor torque only has to compensate the load torque. Then, at time $t = 1.5\,\text{s}$ the current increases due to the load torque increment.

Figure 7 shows the time evolution of the sliding variable. In this figure it can be seen that the system reach the sliding condition ($S(t) = 0$) at time $t = 0.7\,\text{s}$, but the system lost this condition at time $t = 1.5\,\text{s}$ due to the load torque increment which produces an increment in the system uncertainties that could not be compensated by the actual value of the sliding gain.

Figure 8 presents the time evolution of the adaptive sliding gain. The sliding gain starts from zero and then it is increased until its value is high enough in order to compensate for the system uncertainties. Then, after $t = 0.7\,\text{s}$, the sliding gain is remained constant because the system uncertainties remain constant as well. Later at time $1.5\,\text{s}$, there is an increment in the system uncertainties caused by the rise in the load.
torque. Therefore the sliding gain is adapted once again in order to overcome the new system uncertainties. As it can be observed in the figure after this adaptation the sliding gain remains constant again, since the system uncertainties remains constant as well.

It should be noted that the adaptive sliding gain allows to employ a smaller sliding gain, because the sliding gain does not have to be chosen high enough to compensate for all the possible uncertainties that can be appear in the system. In this way in the proposed adaptive scheme the sliding gain will be adapted (if necessary) when a new uncertainty will appear in the system in order to surmount this uncertainty.

V. CONCLUSIONS

In this paper a robust position regulation for an induction motors using an adaptive sliding mode vector control has been presented. The proposed variable structure control incorporates an adaptive algorithm to calculate the sliding gain value. The the sliding gain adaptation, on the one hand avoids the necessity of calculate the upper bound for the system uncertainties, and on the other hand allows to employ a smaller sliding gain in order to overcome the system uncertainties. Therefore, the control signal of the proposed variable structure control schemes will be smaller that the control signals of the traditional variable structure control schemes, because in the last one the sliding gain value should be chosen high enough to overcome all the possible uncertainties that could appear in the system along the time.

Finally, by means of simulation examples, it has been shown that the proposed position control scheme performs reasonably well in practice, and that the position tracking objective is achieved under uncertainties in the system parameters and under load torque variations.

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