Harmonic current injection in multi-phase machines for high specific torque including skin effect

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***Abstract*— The additional degrees of freedom in multi-phase machines have been used for several purposes, including harmonic current injection for torque enhancement. This new approach with non-sinusoidal flux distributions leads to new problems and so conventional assumptions have to be revised. Specifically, when injecting current harmonics, the skin effect becomes more relevant due to the high frequency dependence of the machine parameters. This paper analyzes the effect of parameter detuning due to the skin effect in the presence of current harmonics. Simulated results of the behavior of a current-fed five-phase machine are shown to verify the relevance of the detuned parameters in this application.**

***Key words*— Multi-phase machines, skin effect, current harmonic injection, parameter detuning, induction machines.**

I. Introduction

THE interest in multi-phase machines for high performance applications has been growing in recent years due to their potential advantages over three-phase machines. Among the different benefits, there is a reduction in the torque pulsation and so in the machine's vibrations and noise, a reduction in the rotor harmonic losses due to the cancellation of some time current harmonics, a reduction in the stator copper losses, an improved reliability due to the ability to operate after the loss of a stator phase and a reduction in the inverter phase current for a given power [1]. Specially the last advantages, makes the multi-phase machines more attractive for high-power applications, such as electric vehicles, electric ship propulsion, etc.

Apart from the inherent advantages of multi-phase machines, the degrees of freedom existing when the number of phases is greater than three (only two currents are needed to create a rotating field) have pushed researchers to look for a better use of these possibilities. This new field, has led to independent control of machines through series connection and phase transposition [2], reduction in the currents not involved in the electromechanical energy conversion [3], independent estimation of the stator resistance [4] or current harmonic injection for torque enhancement [5]. The last use of these degrees of freedom and the skin effect analysis for this application is the aim of this paper.

The point of view in this case, is quite different from the conventional one. While the usual goal is to reduce current harmonics and create sinusoidal variables [3], now the focus is changed to take advantage of non-sinusoidal spatial and time harmonics. The basis of the method can be found in [5] where a detailed study for different number of phases is carried out, and the harmonic interaction is studied.

While in a three-phase machine the injection of a third harmonic leads to torque ripple, in a five-phase machine, the interaction between spatial and time harmonics of the same order with concentrated windings, makes the current harmonic to create a field rotating at synchronous speed, and so torque enhancement is achieved without additional ripple. The extra torque is also obtained due to the fact that the flux distribution in the airgap is flattened, so that saturation can be avoided for a wider range.

In the same way, for a seven-phase machine, not only the third, but also the fifth current harmonic can be injected increasing the torque of the machine. Following this, for a nine-phase machine the third, fifth and seventh current harmonic can be injected without torque ripple. The higher the harmonic injected is, the more relevant is the skin effect both in stator and rotor resistances and inductances [6]. Conventional assumption of neglecting this effect can be less justified for this application. The main goal of this work is to analyze the skin effect when injecting current harmonics in a five-phase machine, although the effect would be more relevant for say, asymmetrical nine-phase machines. Since the aim is to have an idea of the relevance of the effect, only open-loop simulations will be carried out, being the control of the machine out of the scope of the paper.

All in all, the relevance of the skin effect, when injecting current harmonics for torque enhancement, will be evaluated for the first time, to the knowledge of the authors. The fact that, when currents with higher frequency appear in the machine, effects dependent on this frequency can be also increased, is pointed out and assessed in the paper.

In section II the machine model is shown focusing on the changes carried out to include both the skin effect and the spatial harmonics. Classical phase variable model is described first to describe then the changes in the matrices to include the spatial harmonics and finally modify the rotor equivalent circuit to account for the deep-bar effect. Section III shows the results considering the cases with and without spatial harmonics and skin effect. Finally, in section IV the most relevant conclusions are pointed out.

II. Machine model

A. Classical phase variable model

The symmetrical five-phase machine shown in Fig.1 is modelled using the general theory of electrical machines [7] considering a phase variable model. The machine is current-fed, and so for the ideal case the stator currents and their derivatives are known. From the voltage rotor equation in matrix form, rotor currents can be obtained:

$$\underline{i}^r_{ph} = -\int \underline{L}_r^{-1}\left(\underline{R}_r \underline{i}^r_{ph} + \underline{i}^s_{ph}\frac{\mathrm{d}\underline{L}_{sr}}{\mathrm{d}t} + \underline{L}_{sr}\frac{\mathrm{d}\underline{i}^s_{ph}}{\mathrm{d}t}\right)dt \quad (1)$$

With stator currents as inputs, the rotor currents can be obtained and then the torque can be calculated using the general matrix expression:

$$T_e = P\ \underline{i}^s_{ph}\frac{\mathrm{d}\underline{L}_{sr}}{\mathrm{d}\theta}\underline{i}^{r\ T}_{ph} \quad (2)$$

where P is the number of pole pairs.

The rotor currents also allows to calculate the rotor flux from the phase variables:

$$\begin{aligned} \underline{\phi}^r_{ph} &= \underline{L}_r \underline{i}^r_{ph} + \underline{L}_{rs}\underline{i}^s_{ph} \\ \underline{i}^s_{ph} &= [i_{as}, i_{bs}, i_{cs}, i_{ds}, i_{es}]^T \\ \underline{i}^s_{ph} &= [i_{ar}, i_{br}, i_{cr}, i_{dr}, i_{er}]^T \end{aligned} \quad (3)$$

Stator voltages can be reconstructed from stator equations for monitoring purposes:

$$\underline{v}^s_{ph} = \underline{R}_s \underline{i}^s_{ph} + \underline{L}_s\frac{\mathrm{d}\underline{i}^s_{ph}}{\mathrm{d}t} + \frac{\mathrm{d}\underline{L}_{sr}}{\mathrm{d}t}\underline{i}^r_{ph} + \underline{L}_{sr}\frac{\mathrm{d}\underline{i}^r_{ph}}{\mathrm{d}t} \quad (4)$$

All the parameter matrices in (1)-(4) are 5x5 matrices, while all variable vectors are of dimensions 5x1. With the torque obtained from (2), and using the mechanical equation, the motor speed can be obtained and the motor model is therefore completed:

$$\frac{P}{J}\left(T_e - T_L\right) = \frac{\mathrm{d}\omega}{\mathrm{d}t} \quad (5)$$

With the previous equations the evolution of relevant variables such as the rotor flux, speed or electrical torque, can be monitored when stator currents are injected.

Using the space vector decomposition approach detailed in [3] for the asymmetrical six-phase machine, voltages and currents can be then characterized by a general vector:

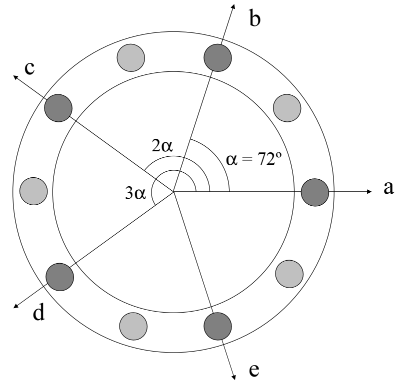

Fig. 1. Stator winding of the asymmetrical five-phase machine.

$$\begin{aligned} S_k(\omega t) = [&\cos(\omega t), \cos(\omega t - \alpha), \cos(\omega t - 2\alpha), \\ &\cos(\omega t - 3\alpha), \cos(\omega t - 4\alpha)]^T \end{aligned}$$

where $k = 0, 1, 2, 3, ...$, denotes the order of time harmonics. When $k = 1$, the fundamental component is obtained, which will be involved in the electromechanical energy conversion. If $\omega t = 0$ and $\omega t = \frac{\pi}{2}$, then two orthogonal vectors are obtained, denoted as d, q:

$$d:\ [1, \cos(\alpha), \cos(2\alpha), \cos(3\alpha), \cos(4\alpha)]^T \quad (6)$$

$$q:\ [0, \sin(\alpha), \sin(2\alpha), \sin(3\alpha), \sin(4\alpha)]^T \quad (7)$$

Currents with a phase displacement equal to the spatial one will generate the rotating field of the machine at the fundamental frequency [3], and they are mapped as d, q components. On the other hand, the third harmonic would be non-electromechanically related for this case and would not generate any additional torque.

B. Concentrated windings phase variable model

If the windings are not distributed but concentrated, then the previous model is no longer valid, since sinusoidal distribution is assumed in the inductance matrices. To account for the spatial harmonics, the previous equations are valid but inductance matrices need to be changed to take into account the new field distribution which has an additional third spatial harmonic. For example, the new stator to rotor mutual inductance matrix can be written as expressed in (8).

In (8) the angles ϕ_i are defined as $\phi_i = \theta + i\alpha$, $\alpha = 72$ degrees is the spatial angle between windings, and θ is the rotor angle. A similar approach is followed for the other matrices so that spatial harmonics are considered. In the same way, if the asymmetrical nine-phase machine is to be modelled, similar matrices would obtained but with different angles ($\alpha = 30$ degrees and expression of ϕ_i differ from the five-phase case).

The new terms in the matrices interact with the third current harmonic that previously did no develop any additional torque, so that in this case third current harmonic is electromechanically related.

$$\underline{L_{sr}} = M \begin{pmatrix} \cos\phi_0 + \frac{\cos 3\phi_0}{3} & \cos\phi_1 + \frac{\cos 3\phi_1}{3} & \cos\phi_2 + \frac{\cos 3\phi_2}{3} & \cos\phi_3 + \frac{\cos 3\phi_3}{3} & \cos\phi_4 + \frac{\cos 3\phi_4}{3} \\ \cos\phi_4 + \frac{\cos 3\phi_4}{3} & \cos\phi_0 + \frac{\cos 3\phi_0}{3} & \cos\phi_1 + \frac{\cos 3\phi_1}{3} & \cos\phi_2 + \frac{\cos 3\phi_2}{3} & \cos\phi_3 + \frac{\cos 3\phi_3}{3} \\ \cos\phi_3 + \frac{\cos 3\phi_3}{3} & \cos\phi_4 + \frac{\cos 3\phi_4}{3} & \cos\phi_0 + \frac{\cos 3\phi_0}{3} & \cos\phi_1 + \frac{\cos 3\phi_1}{3} & \cos\phi_2 + \frac{\cos 3\phi_2}{3} \\ \cos\phi_2 + \frac{\cos 3\phi_2}{3} & \cos\phi_3 + \frac{\cos 3\phi_3}{3} & \cos\phi_4 + \frac{\cos 3\phi_4}{3} & \cos\phi_0 + \frac{\cos 3\phi_0}{3} & \cos\phi_1 + \frac{\cos 3\phi_1}{3} \\ \cos\phi_1 + \frac{\cos 3\phi_1}{3} & \cos\phi_2 + \frac{\cos 3\phi_2}{3} & \cos\phi_3 + \frac{\cos 3\phi_3}{3} & \cos\phi_4 + \frac{\cos 3\phi_4}{3} & \cos\phi_0 + \frac{\cos 3\phi_0}{3} \end{pmatrix} \quad (8)$$

C. Modified phase variable model

The previous model still do not considers the skin effect since the rotor resistance and inductance are not variable. To take into account the deep-bar effect, a network solution based on analytical solutions [6] is considered. The rotor resistance evolution for the case of a bar with a height of 15mm and a width of 5mm is shown in Fig.2.

To include the skin effect into the model, a new model with three branches using a L-shaped rotor equivalent circuit [6] is considered. The chosen circuit with three sections is shown in Fig.3.

This model provides an approximate solution which is valid for the purpose of this paper. The parameters for the new model are:

$$R_{r1} = 40.01\ \Omega \quad R_{r2} = 25.61\ \Omega \quad R_{r3} = 10.55\ \Omega$$
$$Ll_{r1} = 0.016\ \Omega \quad Ll_{r2} = 0.025\ \Omega \quad Ll_{r3} = 0.06\ \Omega$$

while the rest of parameters remain the same.

The equations for the new model are similar to the classical model although each additional branch implies the inclusion of a new equation with 5x5 matrices.

III. Results

This section shows simulated results for different possibilities so that the effect of the skin effect can be evaluated. The different variations that have been considered are:

a) Injecting the third harmonic of the stator current or consider just the fundamental component
b) Consider the skin effect, and so the modified rotor equivalent circuit or just consider fixed rotor parameters according to the classical model
c) Consider a machine with distributed windings and so sinusoidal airgap flux distribution or consider concentrated windings in the machine and include the spatial harmonics

Simulation have been carried out according to the several possibilities with the same inputs. For the input stator currents the rated value has been consider and for the load torque, a load step of twice the rated value is applied at $t = 4s$ during 0.1 s and a final constant load is applied at $t = 5s$ with a value slightly over the rated value.

The speed response can be seen in Fig.4 for the different possibilities showing for all the cases the normal shape for a direct start. From the figure, it is clear that when no third harmonic is injected, the solution with and without concentrated windings coincides. This is due to the orthogonality between the fundamental and third harmonic. Furthermore, if a third harmonic is injected but the windings are considered distributed, the result is also the same, since the current harmonic has no effect since there is not spatial harmonic and orthogonality makes again the harmonic injection not useful.

It is also noticeable that the cases when the harmonic injection is considered are faster than those with the same conditions but without harmonic injection. This is valid both when the skin effect is neglected and when it is not, and it is reasonable since the harmonic injection with concentrated

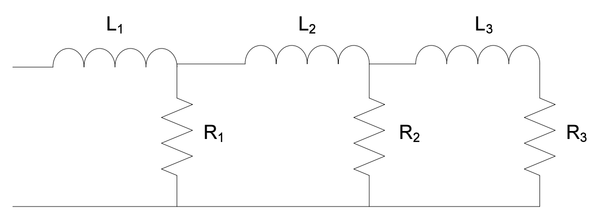

Fig. 3. Rotor equivalent circuit considering three sections

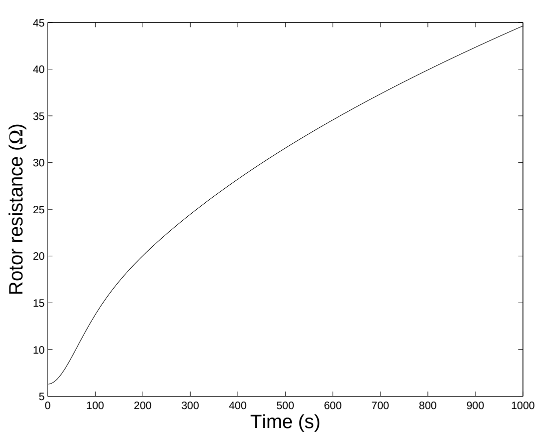

Fig. 2. Rotor resistance evolution as a function of the frequency

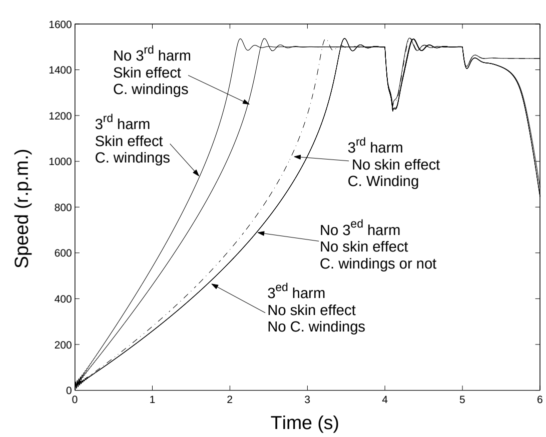

Fig. 4. Motor speed during direct start for the different cases

windings generates an additional torque that, for the same inertia, makes the machine accelerate sooner.

In relation to the skin effect, it is clear that considering the skin effect make the process quicker, due to the fact that initially the rotor frequency is 50 Hz before the synchronous speed is reached, and so the rotor time constant for the case with deep-bar effect is smaller at the beginning, accelerating the process.

At $t = 4s$, a load step is applied for 0.1 seconds making the speed go down and the rotor frequency increase. A detail of the previous figure is shown in Fig.5, and it can be seen that the speed evolution is different if the skin effect is considered or if it is not. For the case with no harmonic injection, the maximum difference is 5 rpm, while for the case with harmonic injection this difference nearly doubles going up to 9 rpm.

A final load is applied at $t = 5s$ with a value slightly over the rated value. This load torque is maintained and so the speed is reduced from synchronous to 1450 rpm approximately for the case when stator current harmonic is applied. Since the torque is over the rated value, for the cases when no harmonic injection is included, the speed can not be maintained since the load torque can not be balanced by the electrical one. Nevertheless, for the case with third stator current harmonic, the additional torque produced by the interaction of the third time and spatial harmonics can balance the load torque and a stable final speed is reached. From this comparison the torque enhancement is verified.

Fig.7 shows the electrical torque for the different cases previously chosen obtaining the normal shape for the direct start with an increase in the torque until the pull out torque is reached. Afterwards, the speed overshoots the synchronous value and the electrical torque goes then to zero since no friction is considered. Finally, when the load step and the final load are applied, the electrical torque is changed due to speed reduction although no stable torque can be achieved when no harmonics are injected.

The stator phase currents for the case with third current harmonic injection is shown in Fig.8. Only the first 20 ms have been considered to be able to se the current shape. This shape of the currents is flattened due to the harmonic inclusion, and this is the same with the flux distribution. This is why even more torque can be obtained from the machine, that is, the harmonic injection avoids up to a certain limit the magnetic saturation. Torque enhancement is achieved because of two phenomena, the additional torque due to the harmonic injection and the possibility to increase the flux without reaching saturation.

In Fig.9 the rotor phase currents flowing through the different branches (i.e. flowing through the thre inductances L_1, L_2

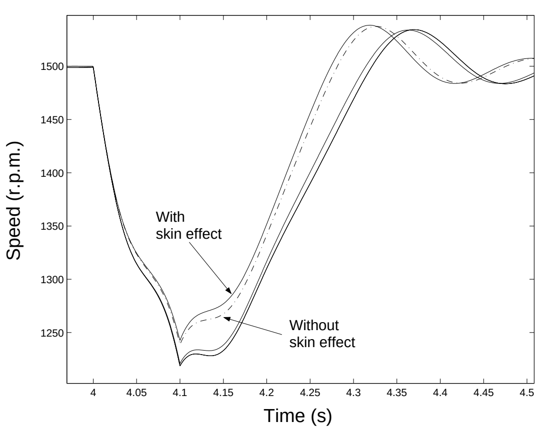

Fig. 5. Detail of motor speed when load step is applied

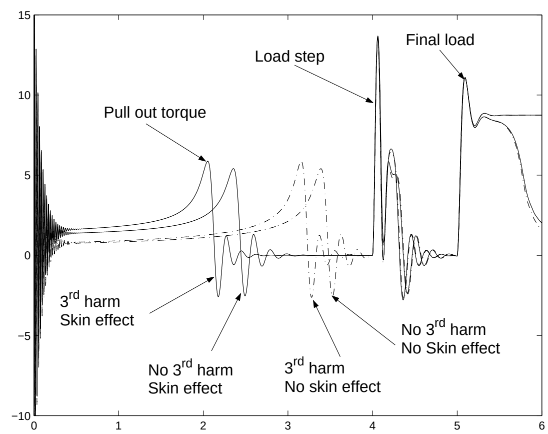

Fig. 7. Torque evolution for the different cases considered

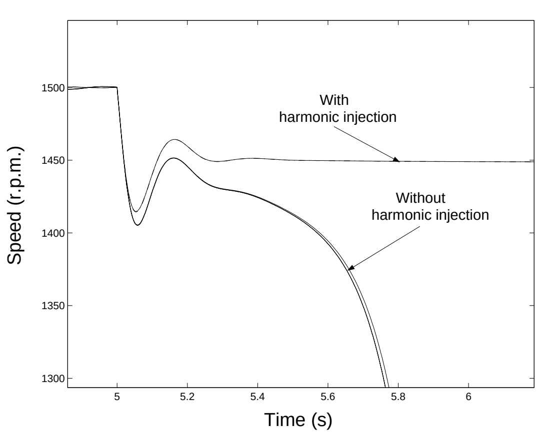

Fig. 6. Detail of motor speed when final load is applied

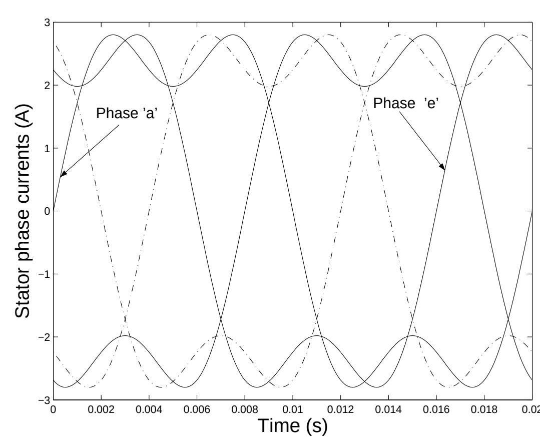

Fig. 8. Stator phase currents

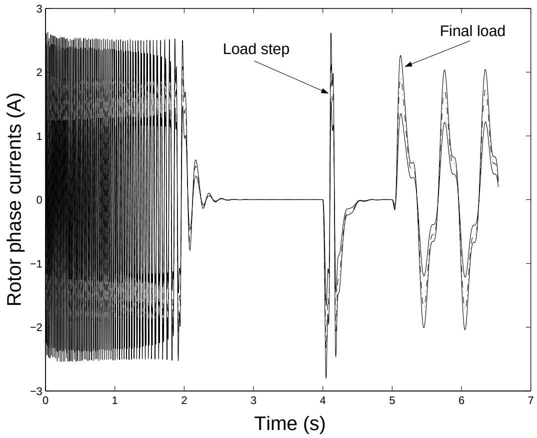

Fig. 9. Rotor phase currents

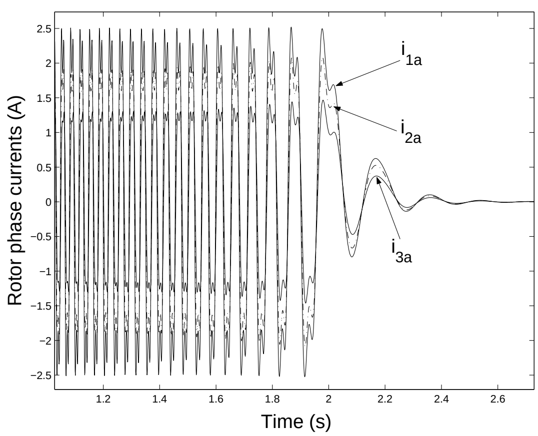

Fig. 10. Rotor phase currents detail

and L_3) i_1, i_2 and i_3 are plotted. The frequency is 50 Hz at $t = 0$, and gradually decreases to zero in steady state with no load. The rotor currents appear again when the load torques are applied due to the speed reduction, providing the necessary electrical torque.It is clear that the rotor currents are no more sinusoidal-shaped wave forms. The three rotor currents are in phase but have different amplitudes.

This fact is more clear in the detail shown in Fig.10, where i_1 is the total rotor current (current flowing through inductance L_1 in Fig.3) while i_2 does not have the contribution of the current flowing in the upper part of the bar (current flowing through ressitance R_1). In this figure it is also clear the fact that the frequency of the electrical rotor current falls to zero when the synchronous speed is reached.

IV. Conclusions

The present work evaluates the impact of the deep-bar effect in a five-phase induction machine performance when injecting third stator current harmonic. A detailed machine modelling is included considering both concentrated windings and skin effect. The approach to model the spatial harmonic is the inductance matrices modification while the deep-bar effect is modelled using a ladder network with a lumped parameter approach. The work takes classical analytical solutions for this effect, and the bars considered are rectangular shaped.

The torque enhancement is fully verified and a comparison is carried out considering several cases regarding the stator winding arrangement, the current source and the inclusion or not of the skin effect.

From this comparative analysis it is clear that this new application, which is of interest in industry applications, implies an increase in the skin effect impact. The speed error is doubled and this can be very important for high performance applications.

It is quite usual to include saturation or temperature variation in high performance drives to improve the drive performance, while deep-bar effect is usually neglected. The results from this study advise to consider this effect when designing high performance drives including harmonic injection, although this effect will be increased or attenuated depending on the number of machine phases and the rotor bar shape.

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