

# Joint wavelet-Fourier analysis of power system disturbances

Juan Carlos Bravo<sup>1</sup>, Dolores Borrás<sup>1</sup>, Manuel Castilla<sup>1</sup>, Antonio López<sup>2</sup>, Jaime Gutiérrez<sup>3</sup> and Juan Carlos Montaña<sup>4</sup>

<sup>1</sup> Dpto. Ingeniería Eléctrica. Escuela Universitaria Politécnica  
Virgen de Africa, 7, 41011-Sevilla, España.  
Tel.:+34 954552831,(2814),(2847), fax:+34 954282777,  
[carlos\\_bravo@us.es](mailto:carlos_bravo@us.es) [borras@us.es](mailto:borras@us.es) [castilla@us.es](mailto:castilla@us.es)

<sup>2</sup> Dpto. Tecnología Electrónica. Escuela Universitaria Politécnica  
Virgen de Africa, 7, 41011-Sevilla, España. [alojeda@us.es](mailto:alojeda@us.es)

<sup>3</sup> Dpto. Física Aplicada III. Escuela Superior de Ingenieros  
Avda. de los Descubrimientos s/n, 41092-Sevilla, España. [jaiben@us.es](mailto:jaiben@us.es)

<sup>4</sup> Consejo Superior de Investigaciones Científicas (CSIC), IRNAS.  
Reina Mercedes S/N. 41080-Sevilla, España. [montano@irnase.csic.es](mailto:montano@irnase.csic.es)

**Abstract.** The Fourier transform usually has been used in the past for analysis of non-stationary signals. Its interest is the knowledge of spectral components existing in a waveform; it doesn't matter the moment where they happen. However, if this information is needed, i.e., if we want to know for transient analyze the s, which spectral component occur at what interval of time, then the Fourier transform is not the right tool. When the time localization of the spectral components is needed, the Wavelet Transform (WT) can be used to obtain the optimal time-frequency representation of the signal. In this paper, a new method of the joint wavelet-Fourier transform has been proposed for detecting, analysing and compacting electrical disturbances. Finally, results of experiments have been included.

**Keywords:** Wavelets, Fourier transform, power quality, harmonic distortion, transients, signal processing.

## 1. Introduction

In order to find out the sources and causes of harmonic distortion, one can detect and localize those disturbances for further classification. Software procedures have been developed for this purpose, such as the FFT [1]; however, due to the amount of stored data and the time of required processing, such procedure is slow and not very efficient.

Continuous and discrete wavelet Transforms (DWT) have been used in analysis of non-stationary signals and many papers such as [3], [4], have been presented proposing the use of wavelets for power systems analysis.

In this environment, power quality analysis strategies have usually been divided into those that address steady-state concerns, such as harmonic distortion, and transient

concerns, like those resulting from faults or switching transient. Technique such as Fourier spectral analysis is often applied to steady-state events while wavelets, classical transient analysis, computer modelling are traditionally used for transient events.

### A. Steady-state events

While there are a few cases where the distortion is randomised, most distortion is periodic, or harmonic. That is, it is nearly the same cycle after cycle, changing very slowly, if at all. The advantage of using a Fourier series to represent distorted waveforms is that it is much easier to find the system response to an input that is sinusoidal. Conventional steady-state analysis techniques can be used. The system is analysed separately at each harmonic. Then the outputs at each frequency are combined to form a new Fourier series, from which the output waveform may be computed, if desired.

Harmonics, by definition, occur in the steady state, and are integer multiples of the fundamental frequency. The waveform distortion that produces the harmonics is present continually or at least for several seconds. Transients are usually dissipated within a few cycles. Transients are associated with changes in the system such as switching a capacitor bank. Harmonics are associated with the continuing operation of a load.

Usually, the higher-order harmonics (above the range of the 25th to 50th, depending on the system) are negligible for power system analysis. While they may cause interference with low-power electronic devices, they are usually not damaging to the power system. It is also difficult to collect sufficiently accurate data to model power systems at these frequencies.

## B. Transient events

Harmonic distortion is blamed for many power quality disturbances that are actually transient. A measurement of the event may show a distorted waveform with obvious high-frequency components. Although transient disturbances contain high-frequency components, transients and harmonics are distinctly different phenomena and are analysed differently. Transient waveforms exhibit the high frequencies only briefly after there has been an abrupt change in the power system. The frequencies are not necessarily harmonics; they are whatever the natural frequencies of the system are at the time of the switching operation. These frequencies have no relation to the system fundamental frequency.

Continuous and discrete wavelet transform (CWT and DWT) have been used in analysis of non-stationary signals and, recently, several papers [6-8] and books [9-11] have been presented proposing the use of wavelets for identifying various categories of power system disturbances. They are able to remove noise and achieve high compression ratios because of the 'concentrating' ability of the wavelet transform. It has proven a powerful signal processing tool in communications in such areas as, data compression, denoising, reconstruction of high-resolution images, and high-quality speech.

The goal of this work is the use of the wavelet analysis as well as the Fourier analysis for a generic signal (voltage or current signals), in transient or steady state situations, for detecting and classifying power quality events. The use of DWT for compressing the disturbed signal before applying FFT permits to get a higher compression rate and a faster signal processing than that obtained using only the conventional DWT/FFT approach.

The proposed method is based on both, the wavelet analysis and the Fourier analysis, as a new hybrid way to study disturbed signals. Definitions and concepts of FFT and WT are introduced in section 2. The proposed method is described in section 3A. Finally, results of simulation are given in section 3B and conclusions in 4.

## 2. The transforms theory

A signal or function  $f(t)$  can often be better analyzed or processed if expressed as a linear decomposition by

$$f(t) = \sum c_l \Psi_l(t) \quad (1)$$

where  $l$  is an integer index,  $c_l$  the real coefficient and  $\Psi_l(t)$  a set of orthonormal functions. One of the key features in signal processing is the choice of a suitable basis to represent in an efficient way the kind of considered signals.

The Fourier Transform (FT) uses basis functions (sine and cosine) to analyze and reconstruct a function. Wavelet approach is more suitable than the Fourier approach when signals are non-stationary. However, in

wavelet analysis, *the scale* that we use to look at data plays a special role. Wavelet algorithms process data at different *scale o resolutions*.

### A. Fourier theory

The Fourier Transform is the main tool for signal spectral decomposition. It represents a signal  $f(t)$  as a superposition of complex exponential of definite frequency  $f$  and infinite time duration, computing the inner products of the signal to be analysed with the complex exponential, i.e.

$$\hat{F}(f) = \int_{-\infty}^{+\infty} f(t) e^{-i2\pi ft} dt, \quad (2)$$

so the original signal can be recovered by means of the inverse formula

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{F}(f) e^{i2\pi ft} df \quad (3)$$

In practice, we never work with the mathematical function  $f(t)$ , but we have partial information concerning it, that is, samples at some regular time interval  $T_s$ . Therefore, our basic input data will be the array

$$f(n) \equiv f(tn) \equiv f(nT_s), \quad n \in 0, 1, \dots, N-1, \quad (4)$$

where  $N$  is the total number of samples of the signal. The quantities  $T_s$  and  $N$  determine the maximum and minimum frequency we are able to resolve. At one hand, according to Shannon's Theorem, we can not go beyond larger frequencies than  $\omega_{max} = \omega_s/2$ , where  $\omega_s = 1/T_s$  is the sample frequency. At the other hand, the minimum frequency will be given by the inverse of the time interval in which we have samples of the signal, that is,  $\omega_{min} = 1/NT_s$ .

First, we proceed to extract the harmonic content using the standard FFT algorithm, that is, we compute the amplitudes  $F_k$  for the definition of the vector  $f(n)$  as a superposition of complex exponential vectors  $e_k(n) = \exp\{i2\pi nk/N\}$ , where  $k \in 0 \dots N-1$ , and  $i$  is the imaginary unit. Each one of this vectors has a definite frequency  $\omega_k = k\omega_{min}$ , so the mapping from  $k$  to  $\omega$  is given by

$$k \rightarrow \omega_k = \frac{k}{NT_s} \quad (5)$$

The equation then reads

$$f(n) = \frac{1}{\sqrt{N}} \Re \left( \sum_{k=0}^{N-1} F_k e^{-i2\pi nk/N} \right), \quad (6)$$

being the expression for each one of the phasors (FFT)

$$F_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N/2-1} f(n) e^{i2\pi nk/N} \quad (7)$$

The component of the signal  $f(n)$  at the frequency  $\omega_k$  is then given by

$$c_k(n) = |F_k| \cos(2\pi\omega_k t_n + \arg(F_k)) \quad (8)$$

We will define fundamental component of  $f(n)$  as the component corresponding to the value of  $k=k_{fun}$  such that  $\omega_{min}k_{fun} = \omega_0$ , where  $\omega_0$  is some pre-fixed frequency. We shall refer to the harmonic *components* of  $f(n)$  as the components of the Fourier transform corresponding to integer multiples of  $k_{fun}$ , i.e, those values of  $k$  belonging to the subset  $\Delta = (k_{fun}, 2k_{fun}, 3k_{fun}, \dots, Mk_{fun})$ , where  $M$  is the highest order for the last harmonic component considered, and it has to be set according to some convention. Finally, we will define the harmonic content of the analysed signal as the superposition of all the previous components, i.e.

$$h(n) = \sum_{k \in \Delta} |F_k| \cos(2\pi\omega_k t_n + \arg(F_k)) \quad (9)$$

### B. Wavelet theory

In the discrete wavelet transform (DWT),  $\Psi_l(t)$  is the mother wavelet, and a great number of different wavelets are used to approximate any given function, such as Daubechies wavelet family.

The DWT is implemented using a Multiresolution Signal Analysis (MRA) algorithm [6] to decompose a given signal into its constituent wavelet subbands or levels (scales) with different time and frequency resolution. Each of the signal scales represents that part of the original signal occurring at that particular time and in that particular frequency band. These individual scales tend to be of uniform width, with respect to the log of their frequencies, as opposed to the uniform frequency widths of the Fourier spectral bands. In the common dyadic decomposition to be used, the scales are separated from adjacent scales by a frequency octave. These decomposed signal posses the powerful time-frequency localization property, which is one of the major benefits provided by the wavelet transform. That is, the resulting decomposed signals can then be analyzed in both the time and frequency domains. The MRA is an adequate and reliable tool to detect signal sharp changes and

clearly display high frequency transient. In order to use the idea multiresolution, we define the *scaling function* and *mother wavelet* respectively by

$$\varphi(t) = \sum h(n) \sqrt{2} \varphi(2t-n) \quad n \in Z \quad (10)$$

$$\psi(t) = \sum g(n) \sqrt{2} \varphi(2t-n) \quad n \in Z \quad (11)$$

where  $h(n)$  are called the scaling function coefficients and  $g(n) = (-1)^n h(1-n)$ .

Sequences  $h(n)$ ,  $n \in Z$  and  $g(n)$ ,  $n \in Z$ , are *quadrature mirror filters* in the terminology of signal analysis. The sequence  $h(n)$  is known as *low pass filter*, while  $g(n)$  is known as *high pass filter*. In this sense, a recorded digitized function  $a_0(n)$ , which is a sampled signal of  $f(t)$ , is decomposed into its smoothed version  $a_1(n)$ , (it contains low frequency components),

$$a_1(n) = \sum_k h(k-2n) \cdot a_0(k) \quad (12)$$

and detailed version  $d_1(n)$ , (it contains higher frequency components),

$$d_1(n) = \sum_k g(k-2n) \cdot a_0(k) \quad (13)$$

using filters  $h(n)$  and  $g(n)$  [6], respectively. This is a first-scale decomposition. The next higher scale decomposition is now based on signal  $a_1(n)$  and so on (Fig. 1). The analysis filter bank divides the spectrum into octave bands. The cutoff frequency for a given level  $j$  is found by

$$fc = \frac{fs}{2^{j+1}} \quad (14)$$

where  $fs$  is the sampling frequency.

So, the original signal  $f(t)$  can be represented approximately as a superposition of *scaling functions*  $\varphi_{j_0,k}(t)$ , and wavelets  $\psi_{j,k}(t)$ .

$$f(t) = \sum_{k=0}^{2^{j_0}-1} a_{j_0,k} \varphi_{j_0,k}(t) + \sum_{j=j_0}^{J-1} \sum_{k=0}^{2^j-1} d_{j,k} \psi_{j,k}(t) \quad (15)$$

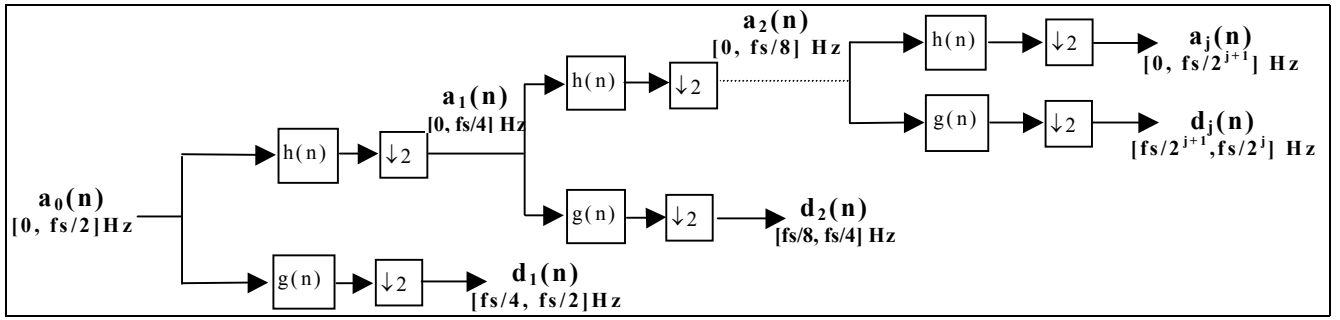


Fig.1: Multiresolution signal decomposition

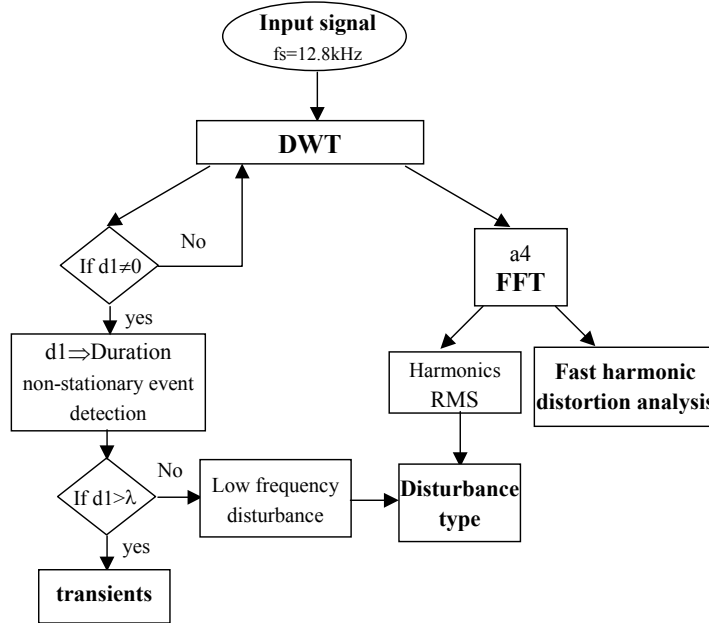


Fig. 2: Flowchart of the proposed method

### 3. Power System Disturbances: detection, location and discrimination

#### A. Method

In this section, we propose an effective method (Fig. 2) to detect, analyse and discriminate power system disturbances using joint wavelet-Fourier decomposition. Our main goal is to perform the desired separated extraction of permanent events (harmonic content), and transient events (random disturbances, such as sags, swell, oscillatory transient, etc.) for a given signal  $f(t)$ . The DWT (Daubechies family Db4) is applied to a digitized function with  $N$  samples, and firstly reaching until level 4, getting signal  $d_1(n)$  and coefficients signal  $a_4$ , according to [5]. The number of samples equals two to the power of the number of levels.

#### 1) Detection and discrimination

The information about the position and length of a power system disturbance is obtained from the detail coefficient  $d_1$ . This signal  $d_1(n)$  is non-zero when a disturbance exists.

After detection of the disturbance, a parallel process begins, on the one hand to compress the input signal applying the FFT over the suitable band to look for stationary events, and on the other a first disturbance discriminations in non-stationary state, based upon the wavelet decomposition characteristics.

Next, a threshold process begins, based at maximum absolute value of  $d_1(n)$  signal. The initial threshold is expressed by

$$\lambda = \eta \cdot \max |a_0(n)| \quad (16)$$

where  $d_1(n)$  is the sampled input signal and  $\eta$  is a parameter that varies between 0.001 and 0.01. Thus,  $\eta=0.005$ , the threshold  $\lambda$  is 0.5% of the maximum value of the  $d_1$  signal. A process of comparison between each point of the  $d_1$  signal and the  $\lambda$  value begins. If any  $d_1(n)$  point is greater than  $\lambda$  a transient event occurs; else a non-stationary low frequency disturbance is present.

When a low frequency disturbance is detected, the mentioned FFT process is needed to distinguish the kind of disturbance: sag, interruption or swell.

## 2) Harmonic analysis and classification

The  $a_4$  coefficient signal is the input data for the FFT analysis. The goal is a faster FFT algorithm because it works over the 1/16 size of the input signal.

The RMS value of the harmonic analysis for this compressed signal enables to discriminate if the disturbance is a sag, an interruption or a swell. If the RMS value remain equal than the fundamental and the  $d_1(n)$  value is non zero, a frequency variation occurs.

In this work we have developed a new approach to detect, classify and analyze various types of power systems disturbances using a joint wavelets-Fourier method. It permits a faster FFT processing and an effective detection and classification of stationary and non-stationary events.

### B. Simulation results

In order to illustrate these ideas, the MATLAB® program is used to calculate the DWT and DFT for two digitized voltage signals, each with 8192 sample points, sampled at  $\omega_s=12.8\text{kHz}$  during total interval  $T$ , equivalent to 32 cycles. In our case, we take as basis harmonic-content: the fundamental component at 50Hz, plus 3rd and 5th harmonic components, with  $220\sqrt{2}$ ,  $\frac{1}{6}220\sqrt{2}$  and  $\frac{1}{10}220\sqrt{2}$  of voltage amplitudes respectively.

Besides this steady-state event, we have also considered a voltage swell, represented by a modulating square wave whose effect is to increase by a 1.2 factor the signal magnitud over an interval of 8 cycles. We have performed numerical simulations with signals that contain steady-state as well as transient disturbances: the first one only present a transient and a momentary swell (fig.3), and the second is the same one with harmonic distortion (fig.4).

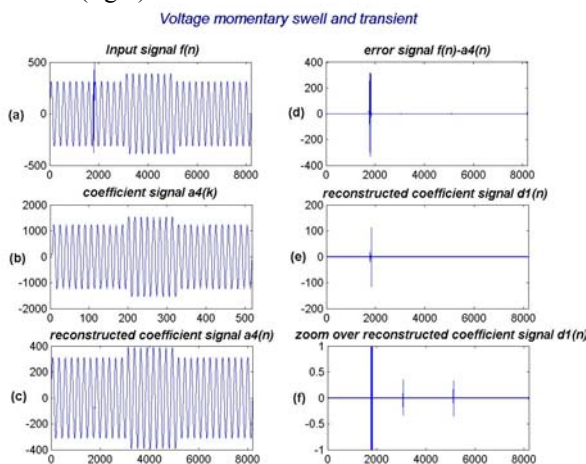


Fig.3 Voltage momentary swell and transient.

Table I  
Number of coefficients

Level	$a_{jk}$	Number of coefficients	$d_{jk}$
1	0 – 3200 Hz	4099	3200–6400 Hz
2	0 – 1600 Hz	2053	1600–3200 Hz
3	0 – 800 Hz	1030	800 – 1600 Hz
4	0 – 400 Hz	518	400 – 800 Hz
5	0 – 200 Hz	262	200 – 400 Hz
6	0 – 100 Hz	134	100 – 200 Hz
7	0 – 50 Hz	70	50 – 100 Hz
8	0 – 25 Hz	38	25 – 50 Hz
9	0 – 12,5 Hz	22	12,5 – 25 Hz
10	0 – 6,25 Hz	14	6,25 – 12,5 Hz
11	0 – 3,125 Hz	10	3,125–6,25 Hz
12	0 – 1,56 Hz	8	1,56–3,125 Hz
13	0 – 0,78 Hz	7	0,78 – 1,56 Hz

For our purpose, first, the DWT (Daubechies family Db4) is applied to a digitized function (a voltage momentary swell and transient) with 8192 samples (fig. 3 (a)). The number of levels is the power of two equal to the number of samples, (in our case  $j=13$ ), and band frequencies corresponding is shown in table I.

A detection process is derived from the  $d_1$  signal (figs. 3(e), 3(f)) and it is fundamental to identify the concrete instant in which the disturbance appears, as well as the time measurement for disturbance duration and, of course, if it's a high or low frequency event.

As a result of the previous development, we achieve the coefficient signals  $a_6(n)$ , which contain the 50Hz component (fig. 4 (c)) and  $a_4(n)$  (fig. 4 (b)) corresponding to low frequency sub-bands where no transient can be found.

By applying the DFT algorithm to the coefficients signal  $a_4(n)$ , we find out an increase of 3.2% in the RMS value over the reference without harmonic distortion. This value is used for classification purpose. Fig 5 shows the harmonics distribution for the coefficients signal  $a_4(n)$ .

The exposed procedure also improves the Fourier analysis because we reduce in a 1/16 the input data for the DFT algorithm, since the coefficients signal used contains only 518 samples instead the 8192 input signal samples, as table I indicates. This work leads us to assure that the joint wavelet-Fourier analysis offers a great potential as a new tool for diagnostic of electrical power systems.

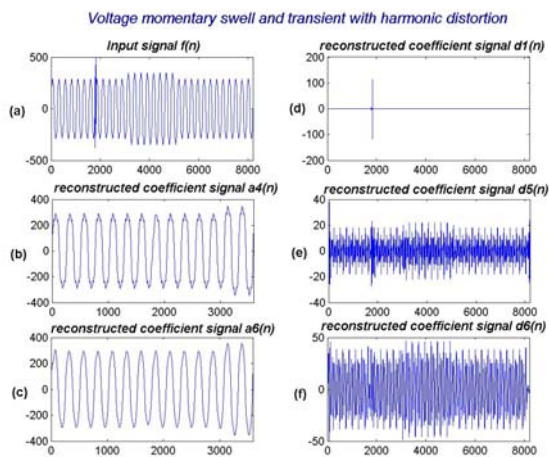


Fig.4 Voltage momentary swell and transient with harmonics distortion

#### 4. Conclusion

DWT is the faster transform for detection and compression purpose in non-stationary states, and the FFT is the best for harmonics location. Both methods (FFT and DWT) have their own limitations in signal disturbance analysis. Our procedure pretends to extract the best of each other for an optimal detection and classification of several disturbances. By this new way, disturbances are detected, analyzed, and classified.

#### 5. References

[1] J.C. Montaño, M. Castilla, J. Gutiérrez, A. López: "Sistema de medida y vigilancia de la calidad del suministro de potencia eléctrica." 5as Jornadas Hispano-Lusas de Ingeniería Eléctrica..Salamanca, Julio 1997.

[2] S.Mallat: "A theory for multiresolution signal decomposition: the wavelet representation." IEEE Trans. on Pattern Anal. and Mach. Intell., Vol. 11, pp.674-693.July 1989.

[3] S.Santoso, E.J.Powers and W.M. Grady: "Power quality disturbances data compression using wavelet transform methods." IEEE Trans. on Power Del. Vol. 12, no. 3. July 1997.

[4] D.Borrás, M. Castilla, N. Moreno and J.C. Montaño: "wavelet and neural structure: a new tool for diagnostic of power system disturbances" IEEE Trans. on Industry App. Vol. 37, no. 1. January 2001.

[5] S. Santoso, P. Hofmann: "Power Quality Assessment via Wavelet Transform analysis" IEEE Trans. on Power Delivery, VOL. 11, nº 2, April 1996.

[6] D.C.Robertson, O.I.Camps, J.S. Mayer and W.B Gish, 'Wavelets and electromagnetic power system transients', IEEE Trans. Power Delivery, vol 11, no. 2, pp. 1050-1058m Apr. 1996

[7] S.Santoso, E.J. Powers, W.M. Grandy and P. Hoffmann, 'Power quality assesment via wavelet transform analysis', IEEE Trans. Power Delivery, vol. 11, pp. 924-930, Apr. 1996.

[8] M.J.T. Smith and S.L. Eddins, 'Analysis/sinthesis techniques for subband image coding', IEEE Trans.

Acoust. Speech, signal Processing, vol.38, pp. 1446-1456, Aug. 1990.

[9] M.V. Wickerhauser, Adapted wavelet analysis from theory to software, IEEE Press, NY, 1993.

[10] J.C. Goswami and A.K. Chan, Fundamentals of Wavelets. Theory, Algorithms and Applications, John Wiley, Inc., NY, 1999.

[11] C.S. Burrus, R.A. Gopinath and H. Guo, Introduction to Wavelets and Wavelet Transforms, Prentice Hall, NJ, 1998.