Measurement of high frequency currents with a Rogowski coil

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Abstract

The main aim of this paper is to expose a simple and low cost device that allows to measure high frequency currents. The Rogowski coil is a flux-to-voltage transducer for non-intrusive current measurement. Its use is usually associated to an external circuit for integrating the measured signal. This paper describes a method for constructing Rogowski coils without using an external integrator.

Keywords: High frequency measurements, Rogowski coil, instrument design, signal integration, voltage transducer.

1. Introduction

The proposed sensor for the measurement of high frequency currents is based on a Rogowski coil [1]. Since 1912, Rogowski coils have been used to detect and measure currents, particularly where large currents like pulsed current mega-amperes in plasma, electron beam, high radiation fields environments and medium voltage cable networks occur. Its application to measure high frequency currents is possible since the development of high bandwidth measurement instrumentation.

A Rogowski coil consists in a toroidal solenoid that encircles a conductor which current is to be measured. The coil is magnetically coupled to the conductor, so that an emf proportional to the variation in time of the current is induced. The advantages of using a Rogowski coil to measure high frequency currents are:

- Non-saturation, because it is air-cored.
- Good linearity, due to the absence of magnetic materials.
- Simple circuitry and low price.
- Non-intrusive, i.e., the coil does not load the circuit carrying the current to be measured under certain conditions of impedance matching.

Rogowski coils are made having no ferromagnetic material in their core. They have two typical kinds of design [2]: those which are wound on a rigid toroidal core and those which are wound on a flexible core. Some authors [3] indicate that those wound on a rigid core provide better accuracy because the second ones are prone to change their characteristics when a turn displacement occurs.

The simplest Rogowski coil design is a single layer winding, as the one shown in Figure 1.

![Figure 1: Single layer winding Rogowski coil [3].](image)

Due to the incremental pitch advancement of the helical winding sum over the circumferential length, any magnetic flux normal to the created loop induces an error voltage into the coil output. To compensate this undesired effect, a one-turn loop is placed inside the helical winding in the opposite direction to the pitch advancement. As it is electrically connected in series with the coil output, a compensation voltage is induced that is equal and of the opposite polarity [3].

2. Operating principle

A Rogowski coil is a current transformer. It consists in an air-cored toroidal coil through which the current to be measured circulates. It is considered as an example of Ampère’s and Fara-
day’s laws. The first one states that the line integral of the magnetic field \( H \) around a single closed path is equal to the current enclosed. Mathematically, it is expressed as follows:

\[
i = \oint \vec{H} \cdot d\vec{l}
\]  

(1)

where:

- \( i \) is the current enclosed,
- \( H \) is the magnetic field, and
- \( dl \) is the infinitesimal element of path length.

The Faraday-Lenz law describes the generation of an emf by a changing magnetic field (Faraday’s law) and the fact that if this emf appears in a closed circuit, the induced current is always in such a direction as to oppose the change that produces it (Lenz’s law).

For an alternating current the voltage output from the coil is given by the rate of change of flux. In an infinitesimal element of path length \( dl \) the magnetic flux linking the section \( dS \) is \( d\Phi \), which is obtained integrating the magnetic field \( \vec{B} \) in the area \( dS \), provided the diameter of the turns is small. The induced voltage in \( dl \) is calculated as the rate of change of the flux:

\[
v_{\text{dl}} = -\frac{d\Phi}{dt} = \frac{d}{dt} \left( \int_S \vec{B} \cdot d\vec{S} \right) = -\frac{d}{dt} \left( \int_S \mu_0 H \cdot d\vec{S} \right) = -\mu_0 A \frac{dH}{dt} \cos \alpha
\]  

(2)

The flux linking the whole coil is given integrating along the coil, knowing that the number of turns per unit of length \( n \) is constant. From there, the total induced voltage can be determined.

\[
v_{\text{coil}} = \int_0^l v_{\text{dl}} \cdot dl = -\mu_0 A n \int_0^l \frac{dH}{dt} \cos \alpha \cdot dl = -\mu_0 A n \frac{d}{dt} \int_0^l H \cos \alpha \cdot dl = -\mu_0 A n \frac{di}{dt}
\]  

(3)

In this way, the current to be measured is related to the induced voltage by a proportionality constant that is the mutual inductance of the coil, \( M \):

\[
v_{\text{coil}} = -M \frac{di}{dt}
\]  

(4)

being \( M = \mu_0 A n \)  

(5)

where \( \mu_0 \) is the air permeability, \( A \) is the turn area and \( n \) the number of turns per unit length.

3. Rogowski coil model

The Rogowski coil is supposed to be equivalent to the circuit shown in Figure 2 [4] where \( R_c \) is the coil resistance, \( L_c \) is its inductance and \( C_c \) is the capacitance. Some authors [5] do not consider the resistance \( R_c \) in their models.

\[\begin{array}{c}
R_c \\
L_c \\
C_c \\
Z \\
\end{array}\]

\[+v_{\text{out}}M \text{d}t\]

\[-v_{\text{coil}}\]

Figure 2: Rogowski coil model [4].

If a measurement impedance \( Z \) is placed in the coil terminals, the transfer function of the model is:

\[
\frac{v_{\text{out}}}{v_{\text{coil}}} = \frac{Z}{L_c Z C_c s^2 + (L_c + R_c Z C_c) s + R_c + Z}
\]  

(6)

4. Integration methods

As it was explained, the induced voltage in the coil is proportional to the current rate of change, i.e., to obtain a voltage signal proportional to the current waveform, integration of \( v_{\text{coil}} \) is required. Several options for integrating were considered:

- mathematical integration, which consists in integrating the signal \( v_{\text{coil}} \) obtained with the oscilloscope or a data acquisition card.
- electronic integration, which consists in integrating with passive integration networks.
- self-integration, through the development of a self-integrating Rogowski coil, as it is explained below.

The last option was chosen and a self-integrating coil was constructed and developed. It is based on using a low enough resistance as terminating impedance \( Z \), so that the transfer function that relates the voltage induced in the coil \( v_{\text{coil}} \) and the voltage measured in the coil terminals \( v_{\text{out}} \) can be simplified as follows:

\[
\frac{v_{\text{out}}}{v_{\text{coil}}} = \frac{R_{\text{out}}}{R_c + s L_c} = \frac{R_{\text{out}}}{R_c + s L_c}
\]  

(7)

When high frequencies currents are to be measured, the term \( s L_c \) will be much bigger than \( R_c \), so that an integrator transfer function is obtained:

\[
\frac{v_{\text{out}}}{v_{\text{coil}}} = \frac{R_{\text{out}}}{R_c + s L_c} = \frac{R_{\text{out}}}{s L_c}
\]  

(8)

Equation 4 gives the voltage induced in the coil \( v_{\text{coil}} \), so that the measured voltage \( v_{\text{out}} \) can be related to the current to be measured \( i \):

\[
v_{\text{out}} = \frac{R_{\text{out}}}{s L_d} \cdot v_{\text{coil}} = \frac{R_{\text{out}}}{s L_c} \cdot M s i = \frac{R_{\text{out}}}{L_c} M i = \kappa i
\]  

(9)

where \( \kappa \) is the coil sensitivity, and it is defined as:
5. Rogowski coil implementation

The Rogowski coil shown in Figure 3 was constructed. A non-ferromagnetic rectangular cross section core was chosen and 12 turns of wire were wound with a one return loop inside the helical winding to compensate any flux parallel to the coil. The geometric characteristics of the constructed coil are shown in Figure 4 and specified in Table 1.

<table>
<thead>
<tr>
<th>Coil parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside diameter, $a$</td>
<td>13 mm</td>
</tr>
<tr>
<td>Outside diameter, $b$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Thickness, $W$</td>
<td>50 mm</td>
</tr>
<tr>
<td>Number of turns</td>
<td>12</td>
</tr>
</tbody>
</table>

Figure 3: Constructed coil.

To obtain the electrical parameters of the Rogowski coil model the impedance analyzer SOLARTRON SI 1260 was employed. This equipment applies a variable voltage to the material and measures the current that passes through it. The ratio of both is the measured impedance. Measuring this ratio at frequencies between 10 Hz and 10 MHz and using a software that comes along with the equipment an adjustment of the parameters of the Rogowski coil model can be obtained. The results are shown in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$</td>
<td>0.2746 ± 0.0004 Ω</td>
</tr>
<tr>
<td>$L_c$</td>
<td>3.083 ± 0.005 μH</td>
</tr>
<tr>
<td>$C_c$</td>
<td>25.2 ± 0.4 pF</td>
</tr>
</tbody>
</table>

Table 2: Parameters obtained with the impedance analyzer.

to obtain a sinusoidal voltage. The current $i$ is circulating through the resistance $R_{cir} = 1$ KΩ while the Rogowski coil encircles the conductor to measure this current.

In order to compare the real current with the measurement of the Rogowski coil, the voltage in the resistance $R_{cir}$ was registered in channels 1 and 2 of the oscilloscope so that the current $i$ could be calculated. In channel 3 of the oscilloscope, the voltage in the terminating resistance $R_{out}$ was recorded.

6. Experimental measurements

Measurements of high frequency sinusoidal waves were made with the measurement layout shown in Figure 5. A metallic box was used in order to filter external high frequency noise. A signal generator was employed.

In Figure 6, the signals obtained for a 5 MHz voltage are shown when measured with two different termi-
nating resistances:

- \( R_{\text{out}} = 1 \Omega \)
- \( R_{\text{out}} = 10 \Omega \)

It can be seen that when using a terminating resistance equal to 10 \( \Omega \), the voltage signal delays about 30\(^\circ\), though its amplitude is much bigger than when using a resistance equal to 1 \( \Omega \). On the other hand, when a terminating resistance of 1 \( \Omega \) is employed, the voltage \( v_{\text{out}} \) is in phase with the current \( i \). That means that \( R_{\text{out}} = 1 \Omega \) is a low enough terminating resistance to get a self-integrating coil at this frequency.

The sensitivity \( \kappa \) is 0.5 V/A and 0.06 V/A for \( R_{\text{out}} = 10 \) and 1 \( \Omega \), respectively, according to equation 10, that states that the sensitivity \( \kappa \) is proportional to the terminating resistance \( R_{\text{out}} \).

When designing a self-integrating coil it has to be noticed that the frequency of the currents to be measured is also a determining factor. For each terminating resistance, the frequency band at which the Rogowski coil behaves as self-integrating has to be calculated. In order to verify this, measurements at three different frequencies (100 KHz, 1 MHz and 10 MHz) with the same terminating resistance (\( R_{\text{out}} = 10 \Omega \)) were made. The results are shown in the Figures 7 to 9.

From these measurements it can be verified that only when the frequency is high enough, the voltage measured in the Rogowski coil terminals is the integral of the voltage induced by the current to be measured. In the figures shown, when measuring a 100 KHz signal the voltage measured \( v_{\text{out}} \) is 90\(^\circ\) dephased from \( i \), which means that the coil is not integrating. In the case of the 1 MHz signal, \( v_{\text{out}} \) is only 45\(^\circ\) dephased, and finally both signals, \( v_{\text{out}} \) and \( i \) are in phase when measuring the 10 MHz signal.

7. Conclusions

A new high-frequency low-levelled pulses measuring probe was constructed and calibrated. With this exceptionally valuable and low priced method fast signals can be measured and recorded in an oscilloscope. A lumped parameter model was exposed and its features were studied so that a method for integrating the induced voltage, without an external circuitry or mathematic posttreatment, could be implemented.

References

Figure 9: Measurement of a 10 MHz sinusoidal wave with a terminating impedance of 10 Ω.