Steady-State Performance Analysis of Brushless DC Propulsion Motor

Xie Wei, Wang Yajin and Hao Ying

Department of Electrical Engineering
Shanghai Maritime University，Shanghai，200135，P. R. China
Phone/Fax：0086-21-58999665　E-mail：weixie@cen.shmtu.edu.cn

Abstract. A mathematical model of brushless DC propulsion motor is established for the steady-state performance analysis. The steady-state currents of the three-phase stator windings can be gained directly according to the symmetrical boundary conditions. The steady-state simulation results are compared with the dynamic ones to validate the correctness of this method.

Key words

BLDC, steady-state, dynamic, propulsion system

1. Introduction

The permanent magnet brushless DC motors (BLDC) are now widely used in the modern motion control systems for their excellent adjustable-speed performance, higher efficiency and smaller size, especially much larger torque at low speed, which is very important for the shipping electric propulsion application (Fig.1).

Because the electric switching elements of the transducer work in the two modes of conduction and turn-off alternately, BLDC actually operates in the quasi-steady state, that is different from the general steady state, and need to be studied deeply [1]. The harmonic analysis is a familiar method that is used to analyze the steady-state performances of electrical devices. As well known, this method is on the base of the superposition principle, the solution results depend on the adopted harmonic orders, and the precision is inadequate for the nonlinear system [2]. If we consider that the steady state is the terminal state of a dynamic process, the steady-state solution can be available from the dynamic simulation. But for steady-state analysis it is necessary to allow the dynamic simulation to run until all transients have died away, so the computing time will become excessive. On the other hand, the dynamic analysis needs precise initial conditions that can't be always obtained easily [3].

In this paper a new method for steady-state performance analysis is put forward on the base of the space-state mathematical model of BLDC. The steady-state solution can be gained directly according to the symmetrical boundary conditions. The validity of this method has been validated by comparing steady-state results with dynamic ones.

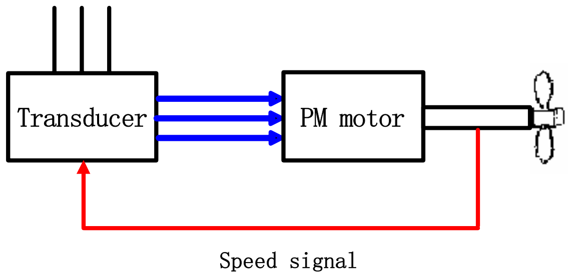


Fig.1. BLDC propulsion system

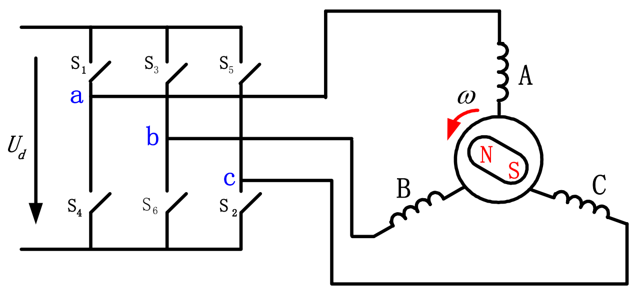


Fig.2. Main circuit of BLDC

2. Mathematical Model

As shown in Fig.2, the voltage equations of the three-phase stator windings are given by

$$\mathbf{u} = \mathbf{R}\mathbf{i} + \mathbf{L}\mathrm{p}\mathbf{i} + \mathbf{e} \qquad (1)$$

where $\mathbf{u}$, $\mathbf{e}$, $\mathbf{i}$, $\mathbf{R}$, $\mathbf{L}$ are the voltage vector, back-EMF vector, current vector, resistance matrix and inductance matrix of the stator windings respectively.

$\mathbf{u} = [\mathbf{u}_a \ \mathbf{u}_b \ \mathbf{u}_c]^{\mathrm{T}}$,

$\mathbf{e} = [e_a \ \ e_b \ \ e_c]^{\mathrm{T}}$,

$\mathbf{i} = [i_a \ \ i_b \ \ i_c]^{\mathrm{T}}$,

$\mathbf{R} = \mathrm{diag}(R, \ R, \ R)$,

$$\mathbf{L} = \begin{bmatrix} L & M & M \\ M & L & M \\ M & M & L \end{bmatrix}.$$

Equ.1 can be written as the standard form of state equation,

$$\mathrm{p}\mathbf{i} = -\mathbf{L}^{-1}\mathbf{R}\mathbf{i} + \mathbf{L}^{-1}(\mathbf{u} - \mathbf{e}) \qquad (2)$$

If the core saturation is omitted, Equ.2 are ordinary differential equations, the solution is shown below

$$\mathbf{i}(t) = \mathbf{i}_s(t) + \mathbf{i}_{\mathbf{t}}(t) \qquad (3)$$

where

$\mathbf{i}_{\mathbf{s}}(t) = \mathbf{R}^{-1}(\mathbf{u} - \mathbf{e})$

$\mathbf{i}_t(t) = \mathbf{C} \cdot \mathbf{E}_s(t) \cdot \mathbf{D}_C$

$\mathbf{C}$ is the eigenvector matrix of the coefficient matrix $-\mathbf{L}^{-1}\mathbf{R}$, $\lambda_1, \lambda_2, \lambda_3$ are the corresponding eigenvalues.

$$\mathbf{E}_{\mathbf{s}}(t) = \mathrm{diag}(e^{\lambda_1 t}, e^{\lambda_2 t}, e^{\lambda_3 t}) \qquad (4)$$

$\mathbf{D}_c$ is called scaling vector that can be gained according to the current symmetry and the boundary conditions.

3. Symmetrical Boundary Conditions

When the three-phase AC circuit operates in the steady state, the half-wave symmetry and three-phase symmetry are both available, that are

$$\begin{cases} i_a(\omega t + \pi) = -i_a(\omega t) \\ i_b(\omega t + \pi) = -i_b(\omega t) \\ i_c(\omega t + \pi) = -i_c(\omega t) \end{cases} \qquad (5)$$

and

$$\begin{cases} i_b(\omega t + 2\pi/3) = i_a(\omega t) \\ i_c(\omega t + 2\pi/3) = i_b(\omega t) \\ i_a(\omega t + 2\pi/3) = i_c(\omega t) \end{cases} \qquad (6)$$

If all the angles in the above equations increase $\pi/3$, we can get

$$\begin{cases} i_b(\omega t + \pi) = i_a(\omega t + \pi/3) \\ i_c(\omega t + \pi) = i_b(\omega t + \pi/3) \\ i_a(\omega t + \pi) = i_c(\omega t + \pi/3) \end{cases} \qquad (7)$$

Combined with the Equ.5, the symmetry relations can be summarized as

$$\begin{cases} i_a(\omega t + \pi/3) = -i_b(\omega t) \\ i_b(\omega t + \pi/3) = -i_c(\omega t) \\ i_c(\omega t + \pi/3) = -i_a(\omega t) \end{cases} \qquad (8)$$

In matrix form,

$$\mathbf{i}(\omega t + \pi/3) = \mathbf{M}\mathbf{i}(\omega t) \qquad (9)$$

where $\mathbf{M}$ is called symmetry coefficient matrix.

$$\mathbf{M} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \qquad (10)$$

It is shown that if the three-phase current waves between $0 \le \omega t \le \pi/3$ are known, the current waves between $\pi/3 \le \omega t \le 2\pi/3$ can be understood easily according to the symmetry relations. The latter currents can always be calculated as this transmissive method. So the solution zone $0 \le \omega t \le \pi/3$ need only to be considered in practice. If $\omega t = 0$, Equ.9 can be expressed as

$$\mathbf{i}_s + \mathbf{C} \cdot \mathbf{E}_s(T_6) \cdot \mathbf{D}_C = \mathbf{M}(\mathbf{i}_s + \mathbf{C} \cdot \mathbf{E}_s(0) \cdot \mathbf{D}_C) \qquad (11)$$

Then the scaling vector $\mathbf{D}_c$ is available,

$$\mathbf{D}_c = (\mathbf{M} \cdot \mathbf{C} - \mathbf{C} \cdot \mathbf{E}_s(T_6))^{-1}(\mathbf{E} - \mathbf{M}) \cdot \mathbf{i}_s \qquad (12)$$

where

$$T_6 = T/6 = \pi/3\omega ,$$

$$\mathbf{E}_\mathbf{s}(0) = \mathbf{E} = \mathrm{diag}(1, 1, 1) .$$

4. Calculation Example

On the base of the method described above, the program for computing the steady-state performance of brushless DC propulsion motor is made, and the steady-state currents of a designed motor are calculated. The Specifications and main parameters of the designed motor are presented in Table I, and the waves of back-EMF are shown in Fig.3.

The calculation results of the steady-state currents are shown in Fig4. In order to validate the correctness of this method, the starting process is also calculated according to the state-space method [4], and the calculation results of the dynamic currents are shown in Fig5. As the steady state is the ultimate objective of the dynamic process, we can get Fig.6 from Fig.5. It is clear that the ultimate dynamic current is very close to the steady-state one.

TABLE I. – Motor Specifications & Parameters

Rated power	50kW
DC Voltage	3000V
Rated speed	100rpm
Number of poles	12
Stator resistance	0.25 Ω
Stator self inductance	35mH
Stator mutual inductance	-4.7mH

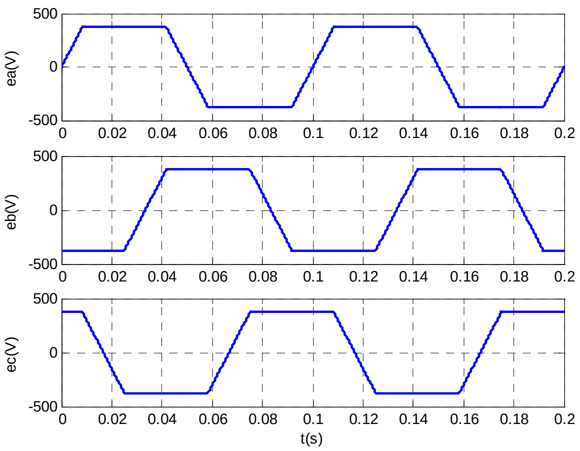
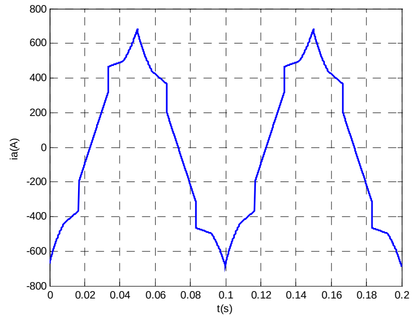
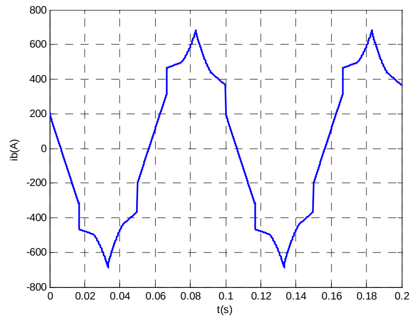


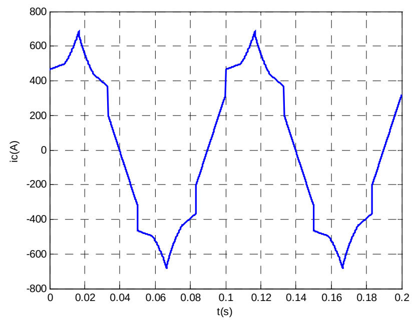
Fig.3. Back-EMF



(a) current i_a

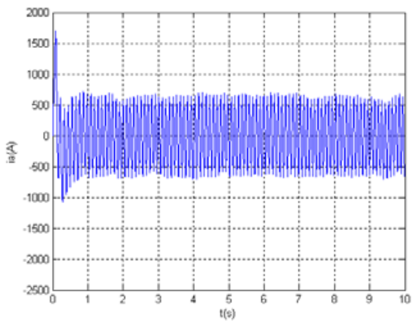


(b) current i_b

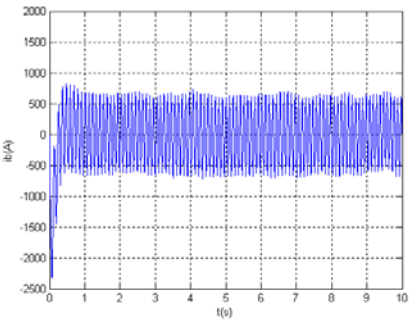


(c) current i_c

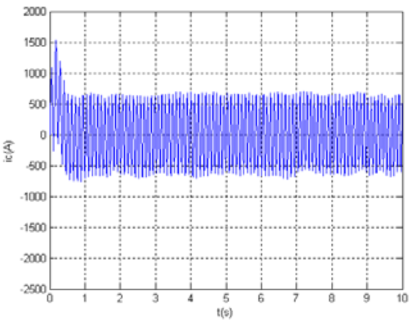
Fig.4. Steady-state current



(a) current i_a

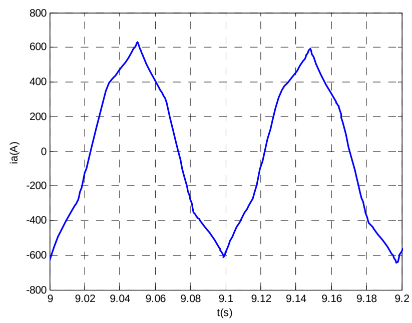


(b) current i_b

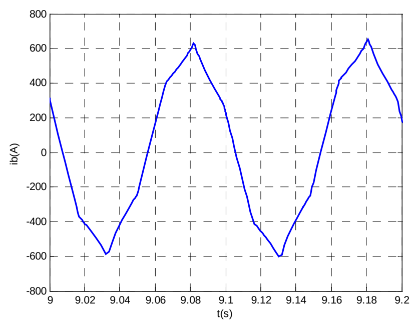


(c) current i_c

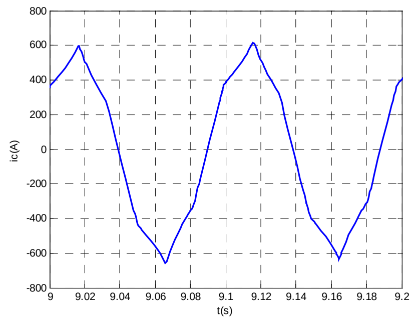
Fig.5. Dynamic current



(a) current i_a



(b) current i_b



(c) current i_c

Fig.6. Ultimate dynamic current

5. Conclusion

A new method for the steady-state performance analysis is described on the base of the state-space mathematical model of the brushless DC propulsion motor. The steady-state solution can be gained according to the symmetrical boundary conditions directly and rapidly. The correctness of this method validated by the corresponding dynamic analysis has shown that it can be used to design and analyze the shipping electric propulsion system.

Acknowledgement

The work of this paper is supported by the sustentation fund of Shanghai education committee and Wuhan marine electric propulsion research institute.

References

[1] A. Hiadis, Li Heming and T. H. Barton, “Time-domain analysis of induction motors with non sinusoidal supplies”, *IEEE IAS Conference Record*, pp.555-564, October 1983.

[2] P. D. Evans and D. Brown, “Simulation of brushless DC drive”, *IEE Proc.*, Vol.137, Part B, No.5, pp.299-308, September 1990.

[3] Y. S. Jeon, et al., “A new simulation model of BLDC motor with real back EMF waveform”, in *Proc. Computers in Power Electronics 2000*, pp.217-220.

[4] Xie Wei, Jing Hongmin and Hao Ying, “Dynamic performance analysis of multi-phase permanent magnet synchronous motor in electric propulsion system”, in *Proc. ICEM2004* (CDROM).