# A new method to obtain multiple load flow solutions

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**Abstract.** This paper presents a new method to obtain multiple load flow solutions efficiently, using a conventional Newton power flow analysis. The method is based on the estimation of initial values to start iterating, taking into account physical considerations such as the load level and area voltage profiles or nodes that can be considered as critical. The proposed method allows to obtain unstable solutions, whether the system is working in a critical situation or not. When the system is heavily loaded, in such a way that it can be considered near to the voltage collapse, reaching the unstable solution, close to the stable one, provides information about the proximity to the voltage collapse. The algorithm presented has been applied to several test systems such as the Ward & Hale 6 bus system, the Klos & Kerner 11 bus system and the IEEE 14 and 118 bus systems.

## Keywords

Multiple solutions, load flow, voltage collapse, voltage instability.

## 1. Introduction

The voltage stability phenomenon related to voltage collapse in power systems is one of the most challenging problems that has been studied over the last decades due to its importance in secure and reliable power system operation. A lot of effort has been devoted to study this phenomenon, as the works found in the related bibliography show.

Although voltage collapse nature is essentially dynamic, it can be treated as a static problem if the parameters of the systems change slowly, for instance, during normal load increase. Therefore, load flow calculation for determining proximity to voltage collapse has been widely used [1].

It is well known that the nonlinear load flow equations can present multiple solutions [2]. Usually, only one of these corresponds to the "stable" system operating point and the others correspond to "unstable" points of equilibrium, that are possible in analytical terms, but they are not feasible steady-state situations in the real power system operation.

As a norm the number of solutions decreases as the system is loaded, in such a way that, in the voltage collapse point neighbourhood, only a very similar pair of voltage solutions remains [3]. This solution pair, the stable operating point and the unstable close to it, can be used to predict proximity to voltage collapse, [4] and [6], or to obtain the loading margin [5]. In this context, it is of great interest to develop methods to locate multiple load flow solutions in a fast and efficient way. Several techniques can be found in the bibliography to deal with this problem. In all of them the computation of unstable solutions requires more analytical effort than with the proposed method.

Two main approaches are mentioned in the bibliography for finding multiple load flow solutions. The first one is based on the optimal multiplier presented in [7] and has been developed in [7]-[10]. In these cases, a good initial estimation of the unstable solutions is used to obtain the multiple solutions. The method proposed in this paper can be included in this approach as a new alternative way to calculate initial estimations of the solutions.

Recently, a second approach is the use of symbolic algebraic techniques. The algebraic method to solve the load flow equations with Gröbner bases always obtains, and with certainty, all the solutions [11], which is a very important feature, but it has only been applied to small systems [12] because the Gröbner bases algorithm is computationally very expensive and the computer quickly runs out of memory even with small scale problems (five or more system buses). Its implementation to solve normal sized systems is not yet mature [13] and much more research is necessary.

On the other hand, the continuation method that traces the P-V curves [14], locates the voltage collapse point as its main objective. This technique can also get the part of the curve P-V corresponding to the unstable solutions. In this sense, although it is not its objective, it may be considered as a technique to find multiple solutions but computationally inefficient since several load flow executions are needed.

## 2. Multiple load flow solutions

The solution set for the power system operating in load conditions far from voltage collapse includes, apart from the stable operation solution, other unstable lower voltage solutions, usually associated to voltage instability, and depending on the system, high voltage solutions but shifted around by 180º, which are unstable solutions usually associated to angle instability.

As the system is loaded and operates in the proximity of voltage collapse, the number of lower voltage solutions decreases and the resultant voltages may be close to those of the stable solution. Generally, the methods that seek multiple solutions get this type of solutions because they are of interest for voltage stability studies.

Analysing the multiple load flow solutions is possible to get a measurement of voltage collapse proximity. In this sense, it is not so important obtaining all the possible solutions as finding the unstable solution that remains in the vicinity of voltage collapse conditions. The proposed method in this paper presents a very reliable way to find the unstable solutions of interest.

## 3. Proposed method

The proposed methods in the bibliography for finding multiple load flow solutions require using a suitable analytical form for the load flow equations and do not attend to electrical aspects of the problem. Those that use a good initial estimation, are originally based on the multiplier theory proposed in [7]. These works are highly dependent on the type of formulation used for the load flow equations. All of them use rectangular coordinates to guarantee better convergence. On the other hand, these approaches can present analytical difficulties in the case of load models different from constant power ones and when the different system limits have to be considered, as it is pointed out in [9].

The new method estimates good initial values for calculating the multiple load flow solutions, performing a linear analysis of load modification. From the stable operation point, the system is linearized, converting the power injected in all the buses, apart from the slack bus, into admittances.

Then, the power system operation is expressed by using nodal equations,

$$\underline{\boldsymbol{I}}_n = \underline{\boldsymbol{Y}}\,\underline{\boldsymbol{U}}_n \tag{1}$$

where $\underline{\boldsymbol{I}}_n$ and $\underline{\boldsymbol{U}}_n$ represent the injected node current and node voltage vectors respectively and $\underline{\boldsymbol{Y}}$ the bus admittance matrix including the loads and generations converted to admittances.

Equation (1) can also be written:

$$\begin{pmatrix} \underline{I}_1 \\ \boldsymbol{0} \end{pmatrix} = \begin{pmatrix} \underline{\boldsymbol{Y}}_{11} & \underline{\boldsymbol{Y}}_{1u} \\ \underline{\boldsymbol{Y}}_{u1} & \underline{\boldsymbol{Y}}_{uu} \end{pmatrix} \begin{pmatrix} \underline{U}_1 \\ \underline{\boldsymbol{U}}_u \end{pmatrix} \tag{2}$$

where $\underline{I}_1$ and $\underline{U}_1$ represent the slack bus injected node current and node voltage respectively and $\underline{U}_u$ the node voltage vector of the rest of the buses.

In this situation, new load admittances are added in buses identified previously as critical. As the system is now linear, the new voltages are:

$$\underline{\boldsymbol{U}}_u = -\underline{\boldsymbol{Z}}_{uun}\,\underline{\boldsymbol{Y}}_{u1}\,\underline{U}_1 \tag{3}$$

where $\underline{\boldsymbol{Z}}_{uun}$ represents the bus impedance matrix for all the buses, apart from the slack bus, modified to include the load admittance increases. The voltages given by (3) are used as initial values of the load flow algorithm.

The authors have adapted the load flow program, developed at the Electrical Department of the E.T.S.I.I. of Madrid, to obtain multiple solutions. However, as a first step, a short-circuit analysis program can be used to prove the effectiveness of the method. In this sense, a short circuit with fault impedance can be considered as a way of loading the system.

The load admittance increase in the critical buses provides lower voltage values than those obtained in the initial operable point and at the critical buses, where the load has been added, the voltage values may be considerably lower than the remaining ones. On the other hand, although the loading conditions are different from the operable stable point, the voltage behaviour pattern and the variable correlation are very similar to the unstable solution of the first type referred above.

Once the critical bus or the critical area have been identified, the only parameter left to define, is the value of the load admittance increase. In case of the choice of initial admittance increase does not provide an unstable solution, the algorithm implemented operates to change this value correctly.

The authors have tested the algorithm on systems of different size, and the corresponding multiple solutions have been obtained easily, taking into account some knowledge of the electrical behaviour of the system. On the other hand, this method does not presents the difficulties and limitations of the analytical methods, mentioned above, based on the search for initial values, since the type of formulation used for the load flow equations is not important and the incorporation of different system limits and load models does not represent any additional difficulty to implement the method. In this sense, the optimum performance of the proposed algorithm only depends on the load flow software used .

As far as computation time is concerned, the number of additional load flow executions required to find alternative solutions is the most important aspect and it is related with the load admittance increase chosen and with the identification of critical buses. A sensitivity or a modal analysis can be performed to locate the weakest buses. In general, 3 or 4 additional executions were enough in all the cases tested by the authors.

The flowchart of the algorithm presented in this paper is shown in Figure 1.

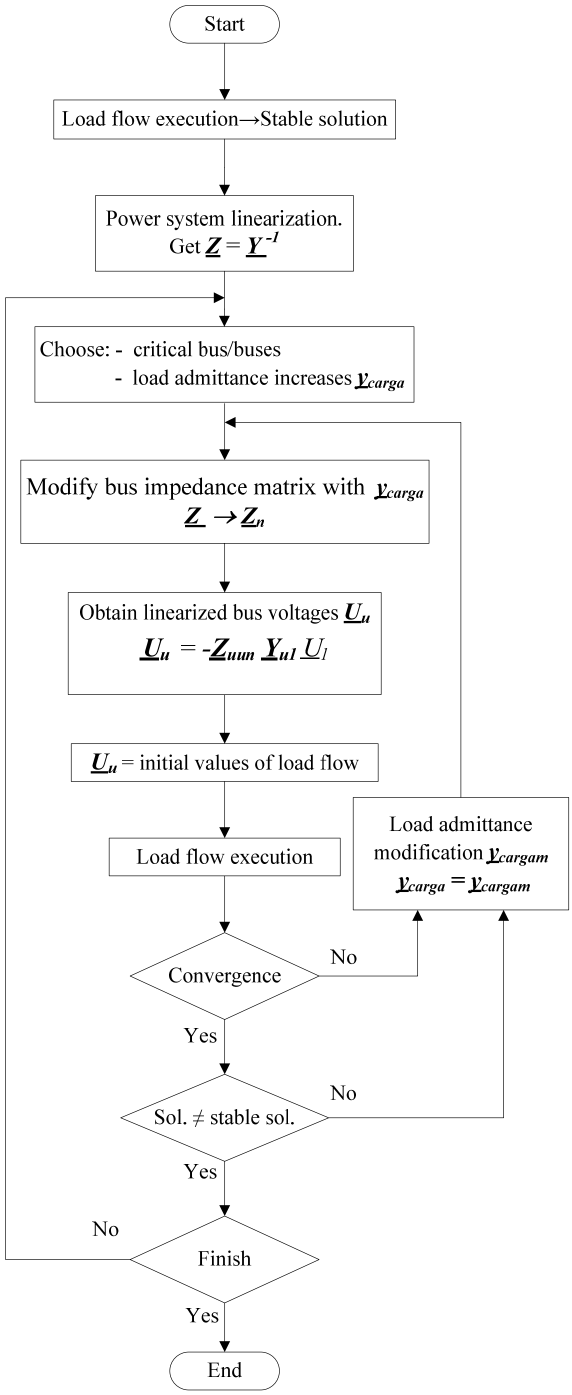


Fig. 1. Flowchart of the proposed algorithm

## 4. Test system results

The proposed method was tested on the Ward & Hale 6 bus system, the heavily loaded Klos & Kerner 11 bus system and the IEEE 14 and 118 bus systems taking into consideration the generation reactive power limits.

In order to perform the load admittance modification efficiently, a sensitivity analysis was applied to detect the weakest buses of the system.

To verify the type of solutions obtained, stable or unstable, modal analysis as described in [15] was used.

### *A. Ward & Hale 6 bus system case*

The case of 6 bus system shown in [16] is analysed under three different load conditions.

1) *Conditions 1 and 2.* In order to demonstrate the proper performance of the proposed method for finding multiple solutions, this test system was analysed using the Gröbner bases to obtain all the analitically feasible solutions. The proposed method provided the same solutions as the ones obtained applying the symbolic algebraic techniques.

The sensitivity analysis performed, in both situations, indicated that bus number 3 was the most critical one.

Condition 1 refers to the 6 bus system when the load demanded at bus 5 is neglected. Two solutions exist under this conditions, a stable solution and an unstable one, that are shown in Table I, and are the same as the ones obtained using the Gröbner bases.

In this case, the unstable solution can be obtained performing a load admittance increase at bus number 3.

TABLE. I. – Multiple solutions for the Ward & Hale 6 bus system neglecting the load demand at bus 5

| Bus | Stable solution | | Unstable solution | |
|---|---|---|---|---|
| | \|U\| p.u. | degrees | \|U\| p.u. | degrees |
| 1 | 1.0500 | 0 | 1.0500 | 0 |
| 2 | 1.1000 | 5.1815 | 1.1000 | -39.3299 |
| 3 | 1.0164 | -9.1484 | 0.1864 | -69.2318 |
| 4 | 0.9511 | -6.6475 | 0.3524 | -18.5612 |
| 5 | 1.0041 | -2.6925 | 0.8139 | -28.7293 |
| 6 | 0.9749 | -6.9367 | 0.6583 | -25.9096 |

Condition 2 refers to the 6 bus system but the new load conditions are: zero load demand at bus 5 and modification of active power demand at bus 3 from 0.55 p.u. to 0.5 p.u. Applying the Gröbner bases algorithm four solutions were encountered, one stable and three unstable. The same three unstable solutions can be found using the proposed method. For instance, among other possibilities, adding a load admittance of 5 p.u. at bus 3 leads to the unstable solution 1, adding a load admittance of 8 p.u. at bus 6 leads to the unstable solution 3 and adding load admittances of 6 p.u. at buses 3 and 6, at the same time, leads to the unstable solution 2.

So far, limits for reactive generation were not taken into account. The method allows for it. For instance, assuming that the maximum reactive generation at bus 2 is 0.4 p.u., adding a load admittance of 5 p.u. at bus 3 leads to the unstable solution 4.

Table II shows these solutions in detail.

TABLE. II. – Multiple solutions for the Ward & Hale 6 bus system without load demand at bus 5 and $P_3$ = 0.5 p.u.

(a)

| Bus | Stable solution | | Unstable solution 1 | |
|---|---|---|---|---|
| | \|U\| p.u. | degrees | \|U\| p.u. | degrees |
| 1 | 1.0500 | 0 | 1.0500 | 0 |
| 2 | 1.1000 | 6.0041 | 1.1000 | -39.1635 |
| 3 | 1.0221 | -8.0330 | 0.1691 | -68.8122 |
| 4 | 0.9561 | -5.8824 | 0.3469 | -17.0010 |
| 5 | 1.0050 | -2.1085 | 0.8123 | -28.4566 |
| 6 | 0.9773 | -6.4284 | 0.6561 | -25.5778 |

(b)

| Bus | Unstable solution 2 | | Unstable solution 3 | |
|---|---|---|---|---|
| | \|U\| p.u. | degrees | \|U\| p.u. | degrees |
| 1 | 1.0500 | 0 | 1.0500 | 0 |
| 2 | 1.1000 | -90.2325 | 1.1000 | -71.7007 |
| 3 | 0.4147 | -61.6316 | 0.5207 | -47.4519 |
| 4 | 0.3778 | -37.0371 | 0.4590 | -31.5931 |
| 5 | 0.4627 | -79.0836 | 0.4276 | -63.9615 |
| 6 | 0.1808 | -86.2591 | 0.1491 | -85.6175 |

(c)

| Bus | Unstable solution 4 Qmax2=0.4 | |
|---|---|---|
| | \|U\| p.u. | degrees |
| 1 | 1.0500 | 0 |
| 2 | 0.7980 $Q_{max}$ | 0.2306 |
| 3 | 0.1928 | -55.4787 |
| 4 | 0.3684 | -13.1184 |
| 5 | 0.6638 | -8.7068 |
| 6 | 0.6246 | -15.0018 |

2) *Condition 3.* The 6 bus system was analysed in the proximity to voltage collapse conditions indicated in [3]. These are, reactive power demand at bus 6 of 1.7 p.u. and a compensation reactance of 1.995 p.u. at the same bus. Two solutions were found under this conditions, one unstable solution closely located to the stable one. The results are shown in Table III.

The weakest bus was the bus 6, after performing the sensitivity analysis.

TABLE. III. – Multiple solutions for the Ward & Hale 6 bus system with $Q_6$ = 1.7 p.u. and $B_6$ = 1.995 p.u.

| Bus | Stable solution | | Unstable solution | |
|---|---|---|---|---|
| | \|U\| p.u. | degrees | \|U\| p.u. | degrees |
| 1 | 1.0500 | 0 | 1.0500 | 0 |
| 2 | 1.1000 | -2.9630 | 1.1000 | -4.9036 |
| 3 | 1.0115 | -12.6597 | 0.9665 | -13.3414 |
| 4 | 0.9405 | -9.7736 | 0.8955 | -10.1625 |
| 5 | 0.9381 | -12.2558 | 0.8606 | -12.8699 |
| 6 | 0.9448 | -12.2141 | 0.8389 | -12.5258 |

In this case, the unstable solution was obtained easily by increasing the load admittance of different buses, for example, buses 6, 5 or 3.

## *B. Klos & Kerner 11 bus system heavily loaded case*

The Klos & Kerner 11 bus system heavily loaded [2] was analysed, but the reactive power injection at bus 11 was varied from 6.3 p.u. to 6.4 p.u., [8]. In this case, two very close solutions exist, as the Table IV indicates.

TABLE. IV. – Multiple solutions for the Klos & Kerner 11 bus system heavily loaded

| Bus | Stable solution | | Unstable solution | |
|---|---|---|---|---|
| | \|U\| p.u. | degrees | \|U\| p.u. | degrees |
| 1 | 1.0500 | 0 | 1.0500 | 0 |
| 2 | 0.7359 | -36.1299 | 0.7139 | -37.2011 |
| 3 | 0.7264 | -56.6308 | 0.7026 | -58.9259 |
| 4 | 0.8271 | -55.1907 | 0.8096 | -57.1957 |
| 5 | 1.0500 | -39.3424 | 1.0500 | -40.9784 |
| 6 | 0.8274 | -53.3018 | 0.8090 | -55.2083 |
| 7 | 0.9090 | -44.5991 | 0.9014 | -45.9704 |
| 8 | 0.9238 | -33.8888 | 0.9182 | -34.9862 |
| 9 | 1.0375 | -15.0459 | 1.0375 | -15.8662 |
| 10 | 0.9682 | -20.2402 | 0.9631 | -20.9377 |
| 11 | 0.8983 | -38.4423 | 0.8823 | -39.6929 |

The sensitivity analysis identifies bus 11 as the most critical one. The unstable solution can be obtained applying a wide load admittance pattern modification. For instance, this solution was found for a load modification at buses 11, 10, 8 or 7.

## *C. IEEE 14 bus system case*

The IEEE 14 bus system considering generation reactive power limits in a heavily loaded situation was studied. The load demand in the PQ buses was proportionally increased, with the initial power factor, as well as the generation active power. The point of collapse, under the previous considerations, is near a power increase of 75%. The results shown in Table V correspond to 60% of power increase.

TABLE. V. – Multiple solutions for the IEEE 14 bus system heavily loaded considering generation reactive power limits

| Bus | Stable solution | | Unstable solution | |
|---|---|---|---|---|
| | \|U\| p.u. | degrees | \|U\| p.u. | degrees |
| 1 | 1.0600 | 0 | 1.0600 | 0 |
| 2 | 0.9817 $Q_{max}$ | -8.0043 | 0.8570 $Q_{max}$ | -8.4381 |
| 3 | 0.8936 $Q_{max}$ | -22.9383 | 0.6652 $Q_{max}$ | -32.2036 |
| 4 | 0.9047 | -17.9422 | 0.6464 | -22.8320 |
| 5 | 0.9169 | -15.0292 | 0.6808 | -17.9296 |
| 6 | 0.9296 $Q_{max}$ | -26.3832 | 0.5092 $Q_{max}$ | -48.7266 |
| 7 | 0.9214 | -24.3091 | 0.5321 | -39.5769 |
| 8 | 0.9652 $Q_{max}$ | -24.3091 | 0.6023 $Q_{max}$ | -39.5769 |
| 9 | 0.8968 | -27.7592 | 0.4466 | -52.5401 |
| 10 | 0.8885 | -28.1411 | 0.4284 | -54.3740 |
| 11 | 0.9022 | -27.5329 | 0.4550 | -52.4134 |
| 12 | 0.9008 | -28.2272 | 0.4437 | -55.7088 |
| 13 | 0.8911 | -28.3867 | 0.4193 | -56.3165 |
| 14 | 0.8592 | -30.3033 | 0.3376 | -66.1180 |

In this case, only one unstable solution was found at the same conditions of generation reactive power limits as in the stable operation.

### D. IEEE 118 bus system case

The IEEE 118 bus system was analysed in a situation close to voltage collapse. From the base case, with a total system load of 3668 MW, the total load and generator participation were increased as proposed in [17] to reach a load of 7446 MW. The inter-area power dispatch was: Main area to area 2 → 102.8 MW and main area to area 3 → 143.8 MW, and the maximum reactive power limit at bus 4 was removed.

The generators reaching their reactive power limits, in the stable solution, are the same as in the reference but for bus 10 generator (Qg10 = 0.9757 p.u.).

An unstable solution was obtained loading bus 2 (Pokagon). Five additional generators reached their maximum reactive power limit (buses 10, 54, 66, 73, 113). The reactive power delivered by bus 4 generator was 7.9267 p.u.

Table VI shows results for three buses in area 2 (2, 3, 9) and two buses in the main area (44, 52).

TABLE. VI. – Multiple solutions for the IEEE 118 bus system for a total system load of 7446 MW, considering all generation reactive power limits, but bus 4

| Bus | Stable solution | | Unstable solution 1 | |
|---|---|---|---|---|
| | \|U\| p.u. | degrees | \|U\| p.u. | degrees |
| 2 | 0.9018 | -30.9860 | 0.8617 | -42.5476 |
| 3 | 0.9017 | -30.4059 | 0.8590 | -42.0490 |
| 9 | 1.0099 | 3.2453 | 0.6610 | 7.5487 |
| 44 | 0.8502 | -15.7173 | 0.8012 | -20.4931 |
| 52 | 0.8954 | -11.8557 | 0.8762 | -14.9061 |

Table VII shows results for other unstable solution. In this case, eight additional generators reached their maximum reactive power limit (buses 40, 42, 54, 61, 66, 73, 113, 116). The reactive power delivered by bus 4 generator was 3.3564 p.u.

From the proximity of the solutions, and from bus 4 generator Q-limit violation, it is possible to get a first idea of the area prone to voltage collapse.

TABLE. VII. – Multiple solutions for the IEEE 118 bus system for a total system load of 7446 MW, considering all generation reactive power limits, but bus 4

| Bus | Stable solution | | Unstable solution 2 | |
|---|---|---|---|---|
| | \|U\| p.u. | degrees | \|U\| p.u. | degrees |
| 2 | 0.9018 | -30.9860 | 0.8853 | -53.8703 |
| 3 | 0.9017 | -30.4059 | 0.8881 | -53.2251 |
| 9 | 1.0099 | 3.2453 | 0.9984 | -18.2662 |
| 44 | 0.8502 | -15.7173 | 0.4898 | -47.0822 |
| 52 | 0.8954 | -11.8557 | 0.5012 | -43.2937 |

## 5. Conclusions

In this paper a new method is proposed to estimate good initial values for calculating the multiple load flow solutions, performing a linear analysis of load modification.

The results show the reliability of the method, since in all the cases tested unstable solutions were achieved, and its robustness, as they can be obtained after a linear analysis of load modification in different buses, not necessarily the weakest or most critical ones.

It is also important to mention that the computation time is reduced if the load modification is performed in the critical buses of the system. In this sense, it is helpful to make a sensitivity analysis to find the weakest buses.

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