Short-Term Stochastic Equilibrium Models for Oligopolistic Electricity Markets

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In liberalized electricity markets the uncertainty associated with markets decission variables has an important effect on firms. The firm's strategic behaviour and its mutual interaction can be modeled by means of equilibrium models. These models combine accuracy at a reasonable computational cost. Equilibrium models with conjectured supply function (CSF) are interesting because they allow to set up rules about the firm's strategic behaviour.

Uncertainty is considered in demand and CSF's slope. There are several methods to take into account uncertainty. In this work scenario tree are used to random variables considered from historical data, probability distributions, parametric families, and so on. This approach has interesting properties like non-anticipativity and usually it allow the use of linear solvers.

The proposed equilibrium model is applied to IEEE-24 as a illustrative case, with 4 competitive firms and linear DC network model. The slope of the conjectured supply function, the inverse demand functions (relationship between price and amount demanded), and the generators costs are taken as linear. The problem is solved in the short term so only one period is considered. In addition a sensitivity analysis of the scenario tree and the model's results to the input data have been performed.

A typical scenario tree in this context has the following structure, as shown in Fig. 1. Where the $P_{i,j}$ are the probability values associated with each branch *j* from each node *i*,

$$\sum_j P_{i,j} = 1, \forall i$$

Some results of this equilibrium model are the probability distribution of the energy price, as shown in Fig. 2 and the probability distribution of the market share of each firm, as shown in Fig. 3.

The mathematical approach consists in put together each of the optimization problems of each firm and the independent system operator (ISO), and the market clearing condition. Each problem has his own objective function and restrictions:

Maximize	Revenues (x)
subject to:	h(x) = 0
	g(x) ≤ 0

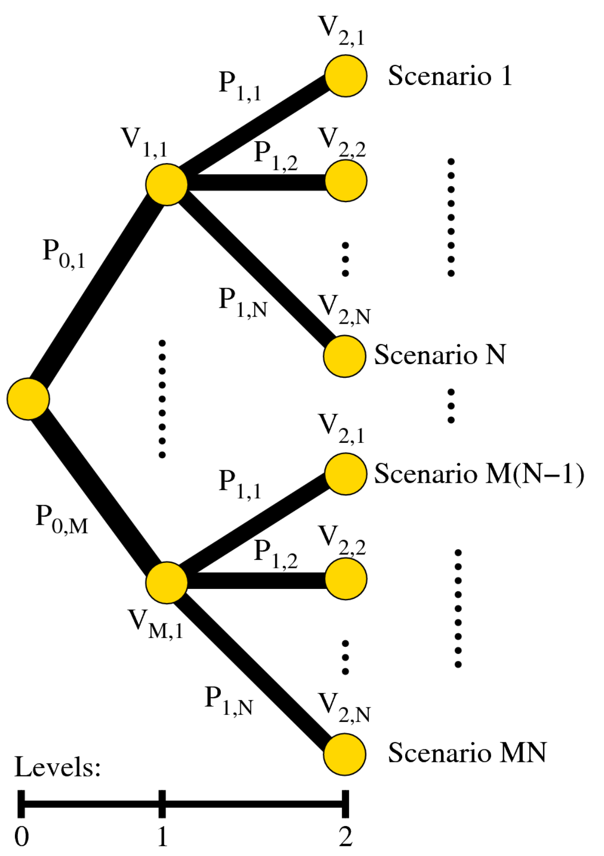

Fig.1 Usual scenario tree used in the equilibrium model.

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For each individual optimization problem its equivalent Karush-Kuhn-Tucker conditions are considered with the market clearing condition all compound the global problem as a mixed complementarity problem, that is solved using GAMS.

Key words: equilibrium model, conjectured supply function, uncertainty, scenario tree, sensitivity analysis.

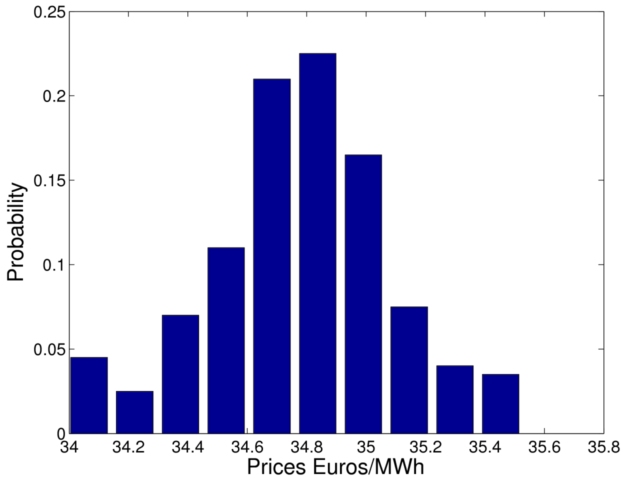

Fig. 2. Price probability distribution.

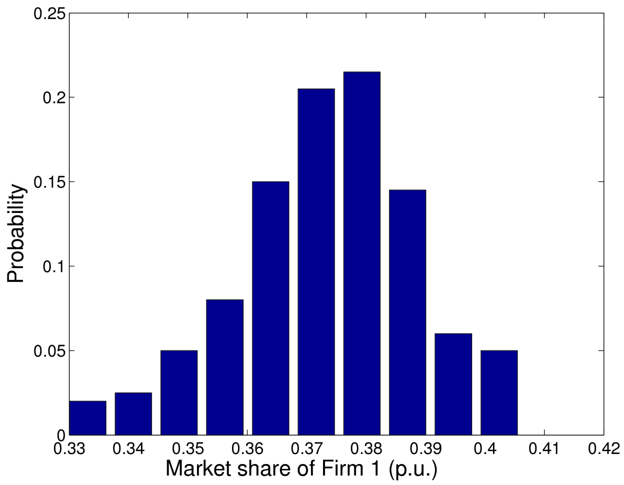

Fig. 3. Market share probability distribution for Firm 1.

References:

[1] B.F. Hobbs, U. Helman, and Jong-Shi Pang. *Equilibrium market power modeling for large scale power systems*. Power Engineering Society Summer Meeting, 2001. IEEE, 1:558{563 vol.1, 2001.

[2] C.J. Day, B.F. Hobbs, and Jong-Shi Pang. *Oligopolistic competition in power networks: a conjectured supply function approach*. Power Systems, IEEE Transactions on, 17(3):597{607, Aug 2002.

[3] H. Heitsch and W. Römisch. *Scenario tree modeling for multistage stochastic programs*. Math. Program. Vol. 118, No. 2, pp. 371-406, 2009.

[4] J. Dupacova. *Stochastic programming: Approximation via scenarios*. In Proceedings of 3rd Caribbean Conference on Approximation and Optimization. EMIS Master Server http://www.emis.de/proceedings/index.html, 1995.