Short-Term Hydro Scheduling: Mixed-Integer Nonlinear Optimisation Methodology

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1. Brief introduction

In this paper, the short-term hydro scheduling (STHS) problem of a head-dependent hydro chain is considered. Hydro plants with only a small storage capacity available are known as run-of-the-river. Due to the reservoirs small storage capacity, the operating efficiency becomes sensitive to the head—head change effect. The cascaded hydraulic configuration coupled with the nonlinear head change effect, augments the problem dimension and the complexity.

The main goal in the profit-based STHS problem is to maximize the value of total hydroelectric generation throughout the time horizon, while satisfying all hydraulic constraints, aiming the most efficient and profitable use of the water [1]. In the STHS problem a time horizon of one day is considered, usually discretized into hourly periods. The STHS problem is treated as a deterministic one. Where the problem includes stochastic quantities such as inflows to reservoirs or energy prices, the corresponding forecasts are used.

Mixed-integer linear programming is becoming often used for STHS [2]-[4], where integer variables allow modelling of start-up costs, which are mainly caused by the increased maintenance of windings and mechanical equipment, and by malfunctions of the control equipment. Hydro scheduling is in nature a nonlinear optimization problem. A nonlinear model has advantages compared with a linear one. A nonlinear model expresses hydroelectric generation characteristics more accurately and the head change effect can be taken into account [1]. However, the nonlinear model cannot avoid water discharges at forbidden intervals. In this paper, we propose a novel mixed-integer nonlinear optimisation methodology to solve the STHS problem.

Key words: Hydro scheduling, head change effect, forbidden zones, mixed-integer nonlinear optimisation.

2. Problem Formulation

A. Objective Function

In this paper, the objective function to be maximized is expressed as

$$\sum_{i=1}^{I}\sum_{k=1}^{K}\left(\lambda_k\, p_{ik}\right)+\sum_{i=1}^{I}\Psi_i\,(v_{iK}) \qquad (1)$$

In (1), the first term is related to the revenues of each plant *i* in the hydro chain.

The last term expresses the future value of the water stored in the reservoirs in the last period *K*.

B. Hydro Constraints

1) Water Balance: The water balance equation for each reservoir is formulated as

$$v_{ik}=v_{i,k-1}+a_{ik}+\sum_{m\in M_i}(q_{mk}+s_{mk})-q_{ik}-s_{ik}\,, \qquad (2)$$

assuming that the time required for water to travel from a reservoir to a reservoir directly downstream is less than the one hour period.

2) Head: The head is considered a function of the water levels in the upstream and downstream reservoirs

$$h_{ik}=l_{f(i)k}(v_{f(i)k})-l_{t(i)k}(v_{t(i)k})\,, \qquad (3)$$

3) Power Generation: Power generation is considered a function of water discharge and hydro power efficiency

$$p_{ik}=q_{ik}\,\eta_{ik}(h_{ik})\,, \qquad (4)$$

Hydro power efficiency is expressed as the output-input ratio, depending on the head. The hydroelectric power generation characteristics can be graphically represented by a family of nonlinear curves, also known as unit performance curves, each curve for a specific value of the head.

4) Water Storage: Water storage has lower and upper bounds

$$\underline{v}_i \le v_{ik} \le \overline{v}_i , \qquad (5)$$

5) Water Discharge: Water discharge has lower and upper bounds

$$u_{ik}\, \underline{q}_i \le q_{ik} \le u_{ik}\, \overline{q}_i , \qquad (6)$$

6) Water Spillage: We consider a null lower bound for water spillage

$$s_{ik} \ge 0 , \qquad (7)$$

Water spillage can occur when without it the water storage exceeds its upper bound, so spilling is necessary due to safety considerations. The initial water storages and inflows to reservoirs are assumed known. Also, the energy prices are considered as deterministic input data for our STHS problem.

3. Mixed-Integer Nonlinear Optimisation

Power generation is considered a nonlinear function of water discharge and water storage, given by

$$p_{ik} = \alpha_i\, \beta_{f(i)}\, q_{ik}\, v_{f(i)k} - \alpha_i\, \beta_{t(i)}\, q_{ik}\, v_{t(i)k} + \chi_i\, q_{ik} \qquad (8)$$

with

$$\chi_i = \alpha_i\, (l^0_{f(i)} - l^0_{t(i)}) + \eta^0_i , \qquad (9)$$

A major advantage of our mixed-integer nonlinear optimisation methodology is to consider the head change effect in a single function (8) of water discharge and water storage that can be used in a straightforward way, instead of deriving several curves for different heads. As a new contribution to earlier studies [1], discontinuous operating regions are also considered using (6).

4. Case Study

The proposed mixed-integer nonlinear optimisation methodology has been applied on one of the main Portuguese cascaded hydro systems.

The realistic hydro chain has three cascaded reservoirs and is shown in Fig. 1.

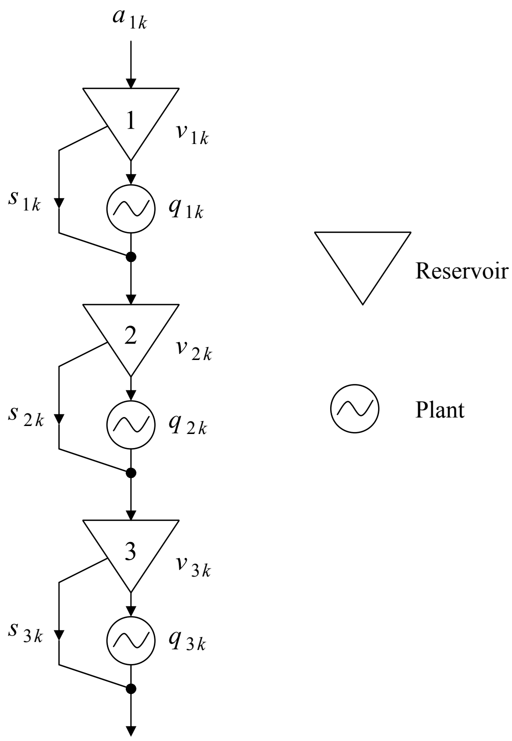

Fig. 1. Cascaded hydro system.

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