Induction heating of cylindrical billets by rotation in uniform magnetic field solved as mechanical transient

M. Donatova[1], P. Karban[1] and I. Dolezel[2]

[1]Department of Theory of Electrical Engineering, Faculty of Electrical Engineering
University of West Bohemia, Univerzitni 26, 306 14 Plzen, Czech Republic
phone: +420 377634647, fax: +420 377634602, e-mail: {mdonat, karban}@kte.zcu.cz

[2]Department of Electrical Power Engineering, Faculty of Electrical Engineering
Czech Technical University, Technicka 2, 166 27 Praha 6, Czech Republic
phone: +420 224353941, fax: +420 233337556, e-mail: dolezel@fel.zcu.cz

Abstract. Induction heating of long cylindrical nonmagnetic billets rotating in uniform magnetic field belongs to progressive techniques of heat treatment of solid metals. The paper presents the complete mathematical model of the process consisting of an integrodifferential equation describing the distribution of eddy current densities in the system, partial differential equation describing the time evolution of temperature field, and a strongly nonlinear ordinary differential equation characterizing the time evolution of rotation. The model is then solved numerically by a combined technique developed by the authors. The theoretical analysis is illustrated on an example whose results are discussed.

Key words

Induction heating, integrodifferential method, finite element method, numerical analysis, electromagnetic field, temperature field.

1. Introduction

Induction heating of nonmagnetic billets is a widely spread industrial technology used mainly for their softening before hot forming. But the conventional heaters made of massive hollow conductor cooled by water exhibit, however, rather low electrical efficiency (about 50–60%). This means that more than 40% of the delivered power is transformed into heat loss in the inductor that is dissipated in the cooling water.

Unfortunately, for induction heating of workpieces of general geometries it is not easy to propose more effective way of the process. Nevertheless, for long cylindrical workpieces an innovative technique was recently introduced consisting in their rotation in a uniform magnetic field produced by either DC currents (Fig. 1) or a system of permanent magnets. In several existing references (see, e.g., [1] and [2]) the steady-state part of the process was analyzed by the finite element method.

The paper offers an alternative approach to mathematical and computer modeling of the process. Its novel feature developed by the authors consists in the description of eddy currents in the system by an integrodifferential equation whose solution directly provides their temporal and spatial distribution in the heated cylinder (thus, without any need to calculate the distribution of the field quantities), which substantially reduces the time of computation and numerical errors. There is also no need to care for the boundary conditions because they are implemented directly in the kernel function of the corresponding integrals. Finally, there is no need to remesh the definition area at every time step. The problem of temperature rise is solved in the quasi-coupled formulation. Respected is also the mechanical transient characterized by the gradual increase of revolutions up to their nominal value.

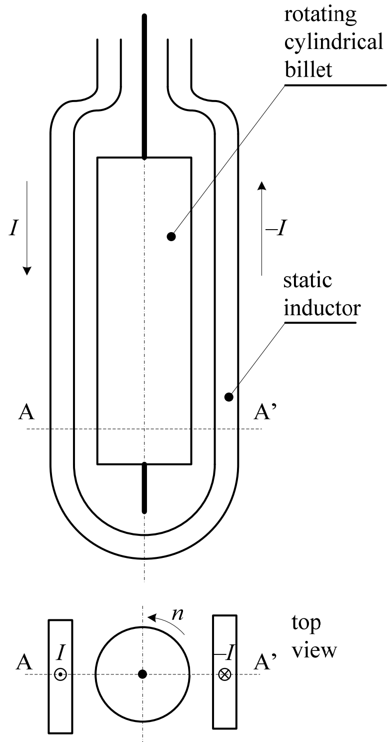


Fig. 1. Induction heating of rotating cylinder by static inductor carrying direct current I

2. Formulation of the problem and its general mathematical model

We first briefly derive the general model of eddy currents and other associated physical quantities (losses and forces) in linear structures with moving elements. The results will be then used for the above model of induction heating.

A. General model of eddy currents in the system

Consider a system containing n nonmagnetic metal bodies $\Omega_j,\ j=1,\dots,n$ (see Fig. 2) whose electrical conductivities are $\gamma_j,\ j=1,\dots,n$. The bodies carry currents $i_j(t),\ j=1,\dots,n$ and each of them can move at a velocity $\boldsymbol{v}_j,\ j=1,\dots,n$. Consider now a reference point $Q_j(\boldsymbol{r}_j)\in\Omega_j$ with the position vector $\boldsymbol{r}_j=\boldsymbol{r}_j(t)$.

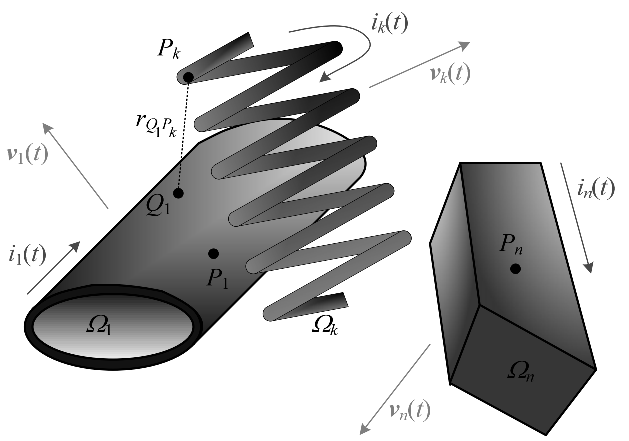


Fig. 2. A system with n electrically conductive current carrying bodies with motion

The value of magnetic vector potential $\boldsymbol{A}$ at reference point $Q_j(\boldsymbol{r}_j)\in\Omega_j$ is given by superposition of components produced by instantaneous current densities in all involved elements and component $\boldsymbol{A}_{j0}$, which is a function of time not known in advance. As the system is linear, we can write

$$\boldsymbol{A}(Q_j,t)=\frac{\mu_0}{4\pi}\cdot\sum_{i=1}^{n}\int_{\Omega_i}\frac{\boldsymbol{J}_i(P_i,t)\cdot\mathrm{d}V}{r_{P_iQ_j}(t)}+\boldsymbol{A}_{j0}(t). \qquad (1)$$

Here, $\boldsymbol{J}_i(P_i,t)$ denotes the vector of total current density at a general integration point $P_i\in\Omega_i$ at time t and $r_{P_iQ_j}(t)$ is the distance between the reference point Q_j and point of integration P_i (for an illustration, Fig. 2 depicts such a distance between points Q_1 and P_k). The total current density $\boldsymbol{J}_i(P_i,t)$ at point P_i consists of the uniform current density $\boldsymbol{J}_{\mathrm{ext},\,i}(P_i,t)$ delivered from the external current source and eddy current density $\boldsymbol{J}_{\mathrm{eddy},\,i}(P_i,t)$.

The Maxwell equation (with respecting motion)

$$\operatorname{curl}\boldsymbol{E}=-\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t}=-\frac{\mathrm{d}}{\mathrm{d}t}(\operatorname{curl}\boldsymbol{A}) \qquad (2)$$

generally provides solution

$$\boldsymbol{E}=-\frac{\mathrm{d}\boldsymbol{A}}{\mathrm{d}t}-\operatorname{grad}\varphi+\boldsymbol{g}(t), \qquad (3)$$

where φ is any scalar function of the coordinates (that can be interpreted as electrical potential) and $\boldsymbol{g}(t)$ is any vector function of time. As no element is supposed to be connected to any supplementary source of the electric field strength (for example, of the thermoelectric origin), we put $\boldsymbol{g}(t)=\boldsymbol{0}$. The electric field strength in all elements is then given only by the source value and time variation of vector potential $\boldsymbol{A}$. Multiplying (3) written for the jth part by the corresponding electrical conductivity γ_j, the vector of eddy current density $\boldsymbol{J}_{\mathrm{eddy},\,i},\ i=1,\dots,n$ can be obtained as

$$\begin{aligned}\boldsymbol{J}_{\mathrm{eddy},\,j}(Q_j,t)&=\boldsymbol{J}_j(Q_j,t)-\boldsymbol{J}_{\mathrm{ext}\,j}(t)\\&=-\gamma_j\cdot\frac{\mathrm{d}\boldsymbol{A}(Q_j,t)}{\mathrm{d}t},\quad j=1,\dots,n.\end{aligned} \qquad (4)$$

Substituting for $\boldsymbol{A}(Q_j,t)$ from (1) we obtain

$$\begin{aligned}&\boldsymbol{J}_j(Q_j,t)+\frac{\mu_0\gamma_j}{4\pi}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=1}^{n}\int_{\Omega_i}\frac{\boldsymbol{J}_i(P_i,t)\cdot\mathrm{d}V}{r_{P_iQ_j}}\\&-\boldsymbol{J}_{\mathrm{ext},\,j}(Q_j,t)+\gamma_j\frac{\mathrm{d}\boldsymbol{A}_{j0}(t)}{\mathrm{d}t}=0,\quad j=1,\dots,n.\end{aligned} \qquad (5)$$

Now the above equation is modified as follows:

$$\begin{aligned}&\boldsymbol{J}_j(Q_j,t)+\frac{\mu_0\gamma_j}{4\pi}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\sum_{i=1}^{n}\int_{\Omega_i}\frac{\boldsymbol{J}_i(P_i,t)\cdot\mathrm{d}V}{r_{P_iQ_j}(t)}\\&+\boldsymbol{J}_{j0}(t)=0,\quad j=1,\dots,n,\end{aligned} \qquad (6)$$

where

$$\boldsymbol{J}_{j0}(t)=-\boldsymbol{J}_{\mathrm{ext},\,j}(t)+\gamma_j\cdot\frac{\mathrm{d}\boldsymbol{A}_{j0}(t)}{\mathrm{d}t} \qquad (7)$$

is (due to the second term on the right-hand side) an unknown time function. This function must be determined indirectly from the condition of the total current that reads

$$\iint_{S_j}\boldsymbol{J}_j(Q_j,t)\mathrm{d}\boldsymbol{S}=i_j(t), \qquad (8)$$

where S_j denotes the corresponding cross section. This relation is applied in such elements where the total current $i_j(t)$ is known.

Let us return to (6) again. This equation may be modified as follows

$$\begin{aligned} & \boldsymbol{J}_j\left(Q_j,t\right)+\frac{\mu_0\gamma_j}{4\pi}\cdot\sum_{i=1}^{n}\int_{\Omega_i}\frac{\frac{\mathrm{d}\boldsymbol{J}_i\left(P_i,t\right)}{\mathrm{d}t}\cdot\mathrm{d}V}{r_{P_iQ_j}(t)} \\ & -\frac{\mu_0\gamma_j}{4\pi}\cdot\sum_{i=1}^{n}\int_{\Omega_i}\frac{\left(\boldsymbol{v}_{ij}(t)\cdot\boldsymbol{r}_{P_iQ_j}(t)\right)\boldsymbol{J}_i\left(P_i,t\right)\mathrm{d}V}{r^3_{P_iQ_j}(t)} \\ & +\boldsymbol{J}_{j0}(t)=0,\quad j=1,\ldots,n, \end{aligned} \tag{9}$$

where $\boldsymbol{v}_{ij}(t)=\boldsymbol{v}_i(t)-\boldsymbol{v}_j(t)$, $\boldsymbol{r}_{P_iQ_j}(t)=\boldsymbol{r}_{P_i}(t)-\boldsymbol{r}_{Q_j}(t)$. The second term on the left-hand side denotes the component of eddy currents due to transformation and the third one another component due to motion.

Now it is still important to determine the specific and total Joule losses p_J and P_J and volume and total Lorentz forces $\boldsymbol{f}_\mathrm{L}$ and $\boldsymbol{F}_\mathrm{L}$. There holds

$$p_{\mathrm{J}j}\left(Q_j,t\right)=\frac{\left|\boldsymbol{J}_j\left(Q_j,t\right)\right|^2}{\gamma_j},\quad P_{\mathrm{J}j}(t)=\int_{\Omega_j}p_j\left(Q_j,t\right)\mathrm{d}V \tag{10}$$

and

$$\begin{aligned} \boldsymbol{f}_{\mathrm{L}j}\left(Q_j,t\right) &= \boldsymbol{J}_j\left(Q_j,t\right)\times\boldsymbol{B}_j\left(Q_j,t\right) \\ &= \boldsymbol{J}_j\left(Q_j,t\right)\times\mathrm{curl}_j\boldsymbol{A}\left(Q_j,t\right), \end{aligned} \tag{11}$$

where the symbol curl_j stands for the operator of circulation that is applied with respect of the coordinates of the reference point Q_j. The total force $\boldsymbol{F}_{\mathrm{L}j}(t)$ acting on the j th element of the system is (after substituting for $\boldsymbol{A}\left(Q_j,t\right)$ from (1))

$$\begin{aligned} \boldsymbol{F}_{\mathrm{L}j}(t) &= \int_{\Omega_j}\boldsymbol{f}_j\left(Q_j,t\right)\mathrm{d}V \\ &= \frac{\mu_0}{4\pi}\int_{\Omega_j}\boldsymbol{J}_j\left(Q_j,t\right)\times\left(\sum_{i=1}^{n}\int_{\Omega_i}\mathrm{curl}_j\left[\frac{\boldsymbol{J}_i\left(P_i,t\right)}{r_{P_iQ_j}(t)}\right]\mathrm{d}V\right)\mathrm{d}V, \end{aligned} \tag{12}$$

where the symbol curl_j denotes again the operator of circulation applied to the coordinates of the reference point Q_j.

B. Nonstationary temperature field

Nonstationary temperature field T in the current carrying parts of the system is described by the heat transfer equation in the form respecting the motion

$$\begin{aligned} \mathrm{div}\left(\lambda\,\mathrm{grad}\,T\right) &= \rho c_\mathrm{p}\frac{\mathrm{d}T}{\mathrm{d}t}-p_\mathrm{J} \\ &= \rho c_\mathrm{p}\left(\frac{\partial T}{\partial t}+\boldsymbol{v}\cdot\mathrm{grad}\,T\right)-p_\mathrm{J}, \end{aligned} \tag{13}$$

where λ is the thermal conductivity, ρ denotes the specific mass, c_p is the specific heat at the constant pressure, symbol $\boldsymbol{v}$ stands for the velocity, and p denotes the specific Joule losses. The physical parameters of materials λ, ρ, and c_p are generally temperature-dependent functions.

Equation (13) has to be supplemented by the boundary conditions that respect the convection and radiation of heat from the body.

$$-\lambda\cdot\frac{\mathrm{d}T}{\mathrm{d}n}=\alpha\left(T-T_0\right)+\sigma C\left(T^4-T_i^4\right), \tag{14}$$

where α is the convective heat transfer coefficient (that is a function of temperature and frequency of rotation), T is the local surface temperature of the billet, σ stands for the Stefan-Boltzmann constant ($\sigma=5.6704\cdot10^{-8}$ kg s^{-3}K^{-4}), C is a constant respecting influences of emissivity, absorption and configuration factors, T_i (simplified) the temperature of the field winding and T_0 the temperature of ambient medium (air).

C. Mechanical transient

The billet is rotated by an asynchronous motor of given torque characteristic $T(\omega)$, where ω denotes the angular frequency. The equation describing the time evolution of ω reads

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t}+D\omega=T(\omega)-T_\mathrm{d}(\omega),\quad\omega(0)=0, \tag{15}$$

where J denotes the moment of inertia of the billet, D is the coefficient of damping and $T_\mathrm{d}(\omega)$ the strongly nonlinear drag torque due to Lorentz forces acting between the primary magnetic field and eddy currents produced in the billet.

3. Modification of the model for the solved arrangement

The solved arrangement is depicted in Fig. 3. For this arrangement it is necessary to appropriately modify equations (9) and (10), and to derive the relation for the drag torque.

Since the arrangement is supposed very long in the axial direction, it can be considered 2D. In such a case the solution can be performed in the polar coordinate system. First, distribution of current densities having the only component in the z direction was derived in [3] or [4] and for the considered case it is given by equation

$$J_{1z}(r,\varphi) = \frac{\mu_0\omega\gamma_1}{2\pi}\cdot\frac{\mathrm{d}}{\mathrm{d}\varphi}\int_{\Omega_1} J_{1z}(P_1)\ln\left(s_{QP_1}\right)\mathrm{d}S + \\ +\frac{\mu_0\omega\gamma_1 J_{2z}}{2\pi}\cdot\frac{\mathrm{d}}{\mathrm{d}\varphi}\int_{\Omega_2}\ln\left(s_{QP_2}\right)\mathrm{d}S + \\ +\frac{\mu_0\omega\gamma_1 J_{3z}}{2\pi}\cdot\frac{\mathrm{d}}{\mathrm{d}\varphi}\int_{\Omega_3}\ln\left(s_{QP_3}\right)\mathrm{d}S, \quad (16)$$

where γ_1 is the electrical conductivity of the billet, s_{QP_1}, s_{QP_2}, and s_{QP_3} are the distances between the reference point Q (φ denoting its angle with respect to the x axis), and general integration points P_1, P_2, and P_3 in regions Ω_1 (billet), Ω_2, and Ω_3 (left and right parts of the field coil), respectively. Finally $J_{2z} = -J_{3z}$ is the current density in the field coil calculated from the current I and cross section of the coil (this density is considered independent of the eddy currents produced in the billet).

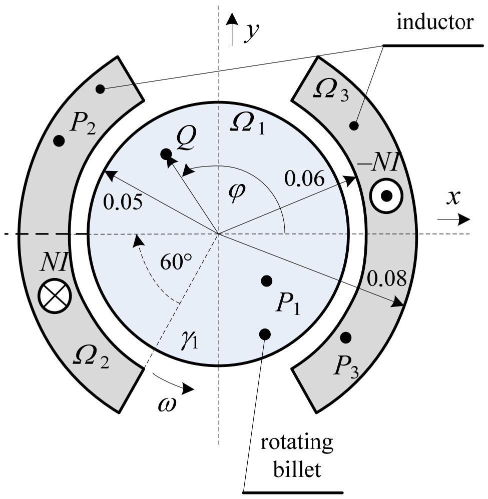


Fig. 3. Detailed view of the solved arrangement

The remaining quantity to be determined is the drag torque $T_\mathrm{d}(\omega)$. First we calculate the elementary force (per unit length) acting at point Q of the billet. Let point Q be described by coordinates r,φ (or coordinates $x = r\cos\varphi, y = r\sin\varphi$), while points P_2 and P_3 by coordinates x_2, y_2 and x_3, y_3, respectively. Now the elementary force $\mathrm{d}\boldsymbol{f}_{\mathrm{L}Q}$ acting at point Q of the billet follows from expression

$$\mathrm{d}\boldsymbol{f}_{\mathrm{L}Q} = \boldsymbol{J}_Q \times \mathrm{d}\boldsymbol{B}_Q\,, \quad (17)$$

where $\boldsymbol{J}_Q$ is the vector of current density at point Q (as previously said, it has only one nonzero component in the z direction) and $\mathrm{d}\boldsymbol{B}_Q$ is magnetic field produced by filaments located at points P_2 and P_3 of both parts Ω_2 and Ω_3 of the inductor. The vector equation (17) may be divided into two component equations

$$\mathrm{d}f_{\mathrm{L}Qx} = -J_{Qz}\cdot\mathrm{d}B_{Qy},\ \ \mathrm{d}f_{\mathrm{L}Qy} = J_{Qz}\cdot\mathrm{d}B_{Qx}\,. \quad (18)$$

The x and y components of magnetic flux density $\mathrm{d}\boldsymbol{B}_Q$ at point Q due to current density at point $P_2 \in \Omega_2$ are given by formulas

$$\mathrm{d}B_{Qx} = -\frac{\mu_0 J_2}{2\pi}\cdot\frac{y-y_2}{\left(x-x_2\right)^2+\left(y-y_2\right)^2} = -\frac{\mu_0 J_2}{2\pi}\cdot\frac{r\cos\varphi-y_2}{\left(r\sin\varphi-x_2\right)^2+\left(r\cos\varphi-y_2\right)^2}\,,$$

$$\mathrm{d}B_{Qy} = \frac{\mu_0 J_2}{2\pi}\cdot\frac{x-x_2}{\left(x-x_2\right)^2+\left(y-y_2\right)^2} = \frac{\mu_0 J_2}{2\pi}\cdot\frac{r\sin\varphi-x_2}{\left(r\sin\varphi-x_2\right)^2+\left(r\cos\varphi-y_2\right)^2} \quad (19)$$

and analogous formulas can be obtained for the contributions due to current density at point $P_3 \in \Omega_3$.

The components of the total local force per unit length $\boldsymbol{f}'_{\mathrm{L}Q}$ can be obtained by integration of (19) over the cross sections S_2 and S_3

$$f'_{\mathrm{L}Qx} = -\int_{S_2} J_{Qz}\cdot\mathrm{d}B_{Qy}\mathrm{d}S_2\,,$$
$$f'_{\mathrm{L}Qy} = \int_{S_2} J_{Qz}\cdot\mathrm{d}B_{Qx}\mathrm{d}S_2\,. \quad (20)$$

The elementary torque $\boldsymbol{t}_Q$ acting at point Q is now given by formula

$$\boldsymbol{t}_Q = \boldsymbol{r}\times\boldsymbol{f}_Q \quad (21)$$

has only one nonzero component in the direction of the z axis

$$t_{Qz} = x\cdot f_{Qy} - y\cdot f_{Qx}\,. \quad (22)$$

The total torque $T'_\mathrm{d}(\omega)$ per unit length is now given by integration of t_{Qz} over Ω_1. The total torque can be then obtained by multiplying the value $T'_\mathrm{d}(\omega)$ by the length of the billet (with some error due to disregarding the front effects).

4. Numerical solution of the model

The model was solved by a code completely developed and written by the authors. The integrodifferential model of eddy currents was solved by a technique similar to the finite elements. The definition area was discretized, the distribution of eddy currents in particular cells of the heated billet was replaced by an appropriate function and their time evolution was solved by the Runge–Kutta method. The nonstationary temperature field (14) was solved by the classical finite element method of the first order that is well known, so that no details will be provided in the paper. As for the movement equation

(15), it was also solved by the Runge–Kutta method, where at each time level the drag torque $T_d(\omega)$ was recalculated appropriately.

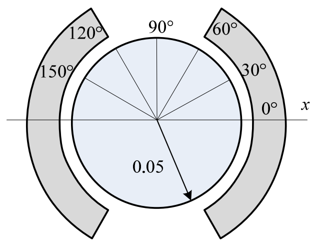
Particular attention was paid to the convergence of results depending on the density of the discretization mesh and time step. As the integrodifferential method works with dense matrices, the highest number of elements in the mesh could not exceed about 10000.

5. Illustrative example

We investigated a lot of examples connected with heating of mostly aluminum billets. The number of the results abounds. For an illustration, we analyzed in details the arrangement whose basic geometry is shown in Fig. 3. Other principal data follow:

- axial length of the billet $L = 0.3$ m,
- mass of the billet $m = 6.362$ kg,
- moment of inertia of the billet $J = 0.00795$ kgm^2,
- field current density $J_{2z} = 2.7285 \times 10^7$ A/m^2,
- coefficient of damping $D = 0.001$ kgm^2/s,
- torque of the induction motor $T(\omega) = 250$ Nm,
- maximum revolutions $n_{max} = 2850$ /min.

The computation took several tens of minutes. Fig. 4 shows the distribution of the specific Joule losses in the steady state (at the maximum revolutions of the drive) produced along the radius of the billet for various radii. It is clear that these losses are produced just in the surface layer of the billet, whose thickness is about 0.015 m (which well corresponds with the depth of penetration).



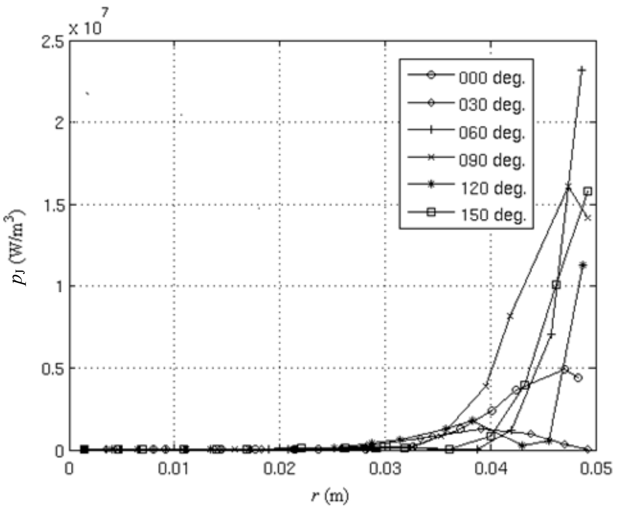


Fig. 4. Distribution of the specific Joule losses along the radius of the billet for various angles

Although the specific Joule losses produced along the radii in Fig. 4 differ from one another, for further computations we can consider their average values during one revolution. The dependence of the average Joule losses p_{Ja} along the radius of the billet is shown in Fig. 5. Even this figure confirms the above conclusion, that is, the specific Joule losses are produced in the layer of thickness corresponding to the depth of penetration.

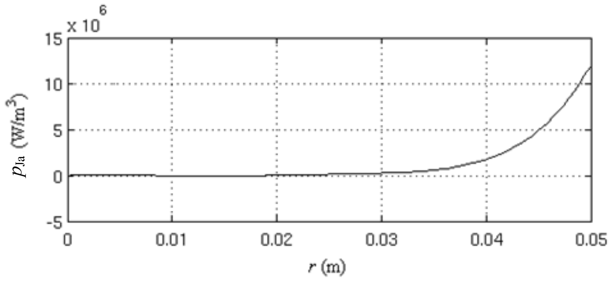


Fig. 5. Distribution of the average specific Joule losses along the radius of the billet

Fig. 6 depicts the dependence of the total drag torque T_d on revolutions. This curve has its maximum for $n \doteq 630$ /min and then it slowly decreases. Fig. 7 shows the time evolution of the revolutions of the billet.

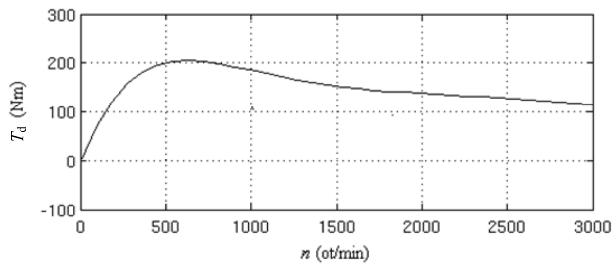


Fig. 6. Dependence of of the drag torque T_d on the revolutions of the billet

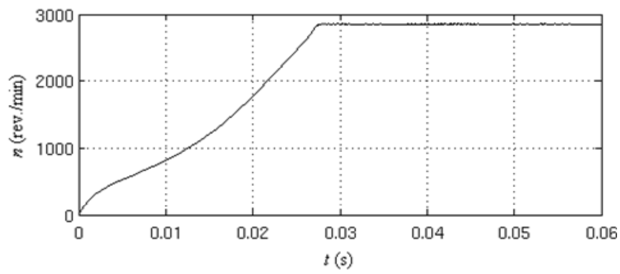


Fig. 7. Time evolution of the revolutions of the billet

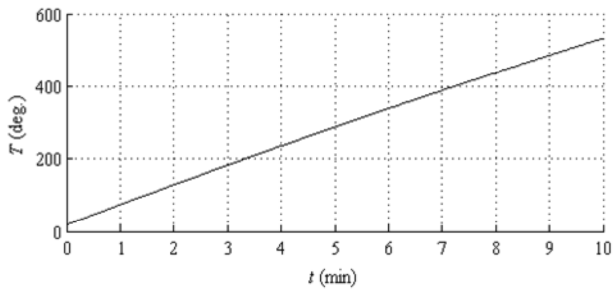


Fig. 8. Time evolution of the average temperature of the billet

Finally, Fig. 8 shows the time evolution of the average temperature of the billet (its distribution in the billet being practically homogeneous due to very good thermal conductivity of aluminum).

6. Conclusion

Some steady-state results were compared with data obtained using professional codes (COMSOL Multiphysics). The accordance was very good (the differences did not exceed about 5%). The future work in the domain will be aimed at the acceleration of the suggested algorithm and evaluation of the inverse possibility of induction heating of an unmoving billet by rotating coils or permanent magnets.

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