

# Critical Clearing Time Evaluation of Power System with UPFC by Energetic Method

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**Abstract.** This paper describes the use of direct method based on energy function type Lyapunov to evaluate the Critical Clearing Time (CCT) of power system subjected to large contingencies. The two axes model of generator with speed and excitation regulation systems is considered in this study.

The power system model takes in consideration the presence of a Unified Power Flow Controller (UPFC). An energy function is constrained for the detailed model with UPFC.

The performance of the Potential Energy Boundary Surface (PEBS) based on the proposed energy function, under different disturbances and loading conditions, is investigated for IEEE 3-machines 9-bus test system. The obtained results show the capability of the proposed approach to precisely evaluate the CCT.

## Key words

Power system, Transient Stability, Critical Clearing Time, Direct Method, Energy Function, Unified Power Flow Controller.

## 1. Introduction

Because of increased system size, greater dependence on controls and deregulated electrical markets, the analysis, required to survive a transient contingencies, is very complex. Additional complicating factors are the operation of the interconnected system with greater interdependence among its members, heavier transmission loadings and the concentration of some large generation units at light loads.

In the modern energy management systems, network application functions are used as valuable tools to balance increasingly stringent operating conditions against economic efficiency. With the growing stress on today's power systems, many utilities increasingly face the threat of transient stability problems. There is a pressing need

for inclusion of on-line dynamic security analysis capabilities in the energy management systems. Dynamic security analysis entails evaluation of the ability of the power system to withstand a set of severe but credible contingencies and to survive transition to an acceptable steady-state condition. Transient stability evaluation which means Critical Clearing Time evaluation is often of concern in these systems.

CCT is defined as maximal fault duration for which the system remains transiently stable. Mathematically, CCT is a complex function of pre-fault system conditions (operating point, topology, system parameters), fault structure (type and location) and post fault conditions that themselves depend on the protective relaying plan employed. It would be highly desirable to define this relation analytically. But, diversity of variables involved makes this task extremely complicated.

In practice, CCT can be obtained in one of two ways: by trial and error analysis of system post disturbance equations [1]-[2] or by integrating fault-on equations and checking the value of Lyapunov energy function until it reaches a previously determined critical level [3]. For the first approach, many integration processes are necessary. But, for the second approach we can evaluate the CCT in just one integration process. The major problem for the second approach is to find an analytical energy function which considers a precise model of generator and the effect of new Flexible AC Transmission System (FACTS) devices added to improve the transient behaviour of power system like UPFC.

In this paper, we deal with the construction of such function. Each generator is represented by a detailed model. This later takes in consideration the regulation systems and the stabilisation devices. The insertion of UPFC is discussed in this paper. An analytical energy

function is presented for this model with UPFC. The PEBS proposed in literature [3] is used to evaluate the CCT.

The IEEE 3-machine 9-bus [4] test systems are used to illustrate the proposed approach in the CCT evaluation. A generalised software tool is elaborated and the simulations performed through the studied systems give promising results.

## 2. System Model

### A. Generator model

A synchronous machine has essentially three fundamental characteristics that need to be represented in the dynamic model. These are the field winding dynamics, the damper winding dynamics, and the shaft dynamics. In that order, the following model includes the field flux-linkage dynamic state variable  $E'_q$ , a single q-axis damper winding-flux linkage state variable  $E'_d$ , the shaft-position state variable relative to synchronous speed  $\delta$ , and the shaft-speed state variable  $\omega$ . The model indexes the equations for a system with  $n$  synchronous machines. The  $i^{\text{th}}$  generator is represented by the following mechanical and electrical first order differential equations:

$$\frac{d\delta_i}{dt} = \omega_i \quad (1)$$

$$\frac{d\omega_i}{dt} = \frac{1}{M_i}(p_{mi} - p_{ei}) \quad (2)$$

$$\frac{d\dot{e}_{di}}{dt} = \frac{1}{T_{d0i}}(-\dot{e}_{di} + (x_{qi} - x_{qi})i_{qi}) \quad (3)$$

$$\frac{d\dot{e}_{qi}}{dt} = \frac{1}{T_{d0i}}(-\dot{e}_{qi} + e_{exi} - (x_{di} - x_{di})i_{di}) \quad (4)$$

The first and second equations are Newton's second law for the dynamics of the rotating shaft. The third equation is the dynamic model that represents Faraday's law for the q-axis damper winding. The fourth equation is the dynamic model that represents Faraday's law for the field winding, and  $e_{ex}$  is the field-winding input voltage. This field voltage is taken either as a constant or a dynamic state variable in the exciter model.

For faster changes in the conditions external to the synchronous machine, this model is the more appropriate.

### B. Speed Regulation Model

When a short circuit takes place near generator station the electrical power output is substantially reduced. The excess mechanical power will be a significant power mismatch across the machine that goes into increasing the kinetic energy of the rotating masses leading to acceleration of the machine. Reduction in mechanical power can be achieved by controlling the steam turbine input with the valve position. The  $i^{\text{th}}$  generator speed governor model is giving by the following first order differential equation:

$$\frac{dp_{mi}}{dt} = \frac{1}{T_{vi}} \left( -p_{mi} + p_{mrefi} - K_{vi} \left( \omega_i + \sigma_i \frac{d\omega_i}{dt} \right) \right) \quad (5)$$

$$P_{mi} = P_{mmai} \quad \text{for} \quad P_{mi} \geq P_{mma}$$

$$P_{mi} = P_{mmii} \quad \text{for} \quad P_{mmii} \geq P_{mi}$$

### C. Excitation Regulation Model

The basic function of an excitation system is to provide a direct current to the synchronous machine field winding. In addition, the excitation system performs control and protective functions essential to the secure operation of the system by controlling the field voltage and hence the field current to be within acceptable levels under different operating conditions. The control functions include the control of voltage and reactive power flow, thereby enhancing power system stability. In the present paper, the excitation system is modelled by the following first order differential equation:

$$\frac{de_{exi}}{dt} = \frac{1}{T_{ei}} \left( -e_{exi} + e_{ex0i} + K_{vi} (v_{refi} - v_i + u_i(t)) \right) \quad (6)$$

$$e_{exi} = E_{xmai} \quad \text{for} \quad e_{exi} \geq E_{xmai}$$

$$e_{exi} = E_{xmi} \quad \text{for} \quad E_{xmi} \geq e_{exi}$$

### D. Network model

During transients, the generator terminal current algebraic variables,  $I_{di}$  and  $I_{qi}$ , are related to the generator terminal voltage,  $v_i$ , and the dynamic states through the following stator algebraic equation [5]:

$$i_{di} = G_{ij} \dot{e}_{di} + B_{ij} \dot{e}_{qi} + \sum_{j=1(\neq i)}^n (\dot{e}_{dj} F_{G+B}(\delta_{ij}) + \dot{e}_{qj} F_{G-B}(\delta_{ij})) \quad (7)$$

$$i_{qi} = G_{ij} \dot{e}_{qi} - B_{ij} \dot{e}_{di} + \sum_{j=1(\neq i)}^n (\dot{e}_{qj} F_{G-B}(\delta_{ij}) - \dot{e}_{dj} F_{G+B}(\delta_{ij})) \quad (8)$$

$$v_{di} = \dot{e}_{di} - x_{qi} \dot{i}_{qi} \quad (9)$$

$$v_{qi} = \dot{e}_{qi} + x_{di} \dot{i}_{di} \quad (10)$$

$$v_i = \sqrt{v_{di}^2 + v_{qi}^2} \quad (11)$$

The electric power of the  $i^{\text{th}}$  machine is given by:

$$p_{ei} = G_{ii} (\dot{e}_{di}^2 + \dot{e}_{qi}^2) + \sum_{j=1(\neq i)}^n (C_{ij} F_{G+B}(\delta_{ij}) - D_{ij} F_{G-B}(\delta_{ij})) \quad (12)$$

With:

$$F_{G+B}(\delta_{ij}) = G_{ij} \cos(\delta_{ij}) + B_{ij} \sin(\delta_{ij}) \quad (13)$$

$$F_{G-B}(\delta_{ij}) = B_{ij} \cos(\delta_{ij}) - G_{ij} \sin(\delta_{ij}) \quad (14)$$

$$C_{ij} = \dot{e}_{di} \dot{e}_{qj} + \dot{e}_{qi} \dot{e}_{dj} \quad (15)$$

$$D_{ij} = \dot{e}_{di} \dot{e}_{qj} - \dot{e}_{qi} \dot{e}_{dj} \quad (16)$$

$G_{ij}$  and  $B_{ij}$  are the real and imaginary parts of the elements  $Y_{rij}$  of the network admittance matrix reduced to generator buses. The loads are modelled as constant admittance's.

$$Y_{rij} = G_{ij} + jB_{ij} \quad (17)$$

$\omega, \Delta\omega$	speed and speed deviation respectively
$\delta$	torque angle
$M$	inertia constant
$P_m$	turbine mechanical power
$P_e$	generator electric power
$i_d, i_q$	d-axis and q-axis component of stator currents
$v_d, v_q$	d-axis and q-axis component of terminal voltage
$v$	terminal voltage
$x_d, x_q$	direct and quadratic synchronous reactance
$x'_d, x'_q$	direct and quadratic transient reactance
$T'_{d0}$	direct transient time constant
$T'_{q0}$	transversal transient time constant
$e'_d, e'_q$	direct and transversal transient <i>f.e.m</i>
$e_{ex}$	excitation voltage
$e_{ex0}$	excitation voltage initial value
$e_q$	permanent <i>f.e.m</i>
$K_e, T_e$	voltage regulator gain and time constant respectively
$E_{xma}, E_{xmi}$	maximal and minimal excitation voltage
$K_v, T_v$	speed regulator gain and time constant respectively
$\sigma$	acceleration action parameter
$P_{mma}, P_{mmi}$	maximal and minimal mechanic power
$P_{mref}$	reference mechanic power

### 3. System Model with UPFC

FACTS controllers are a family of electronic controllers used to enhance power system performance [6]. Certain FACTS controllers have already been applied and others are under development like the UPFC.

Most of works on the transient energy function method are focused to determine the first swing stability limit of power system without any FACTS devices. However, FACTS devices are now increasingly used in power system to improve transient stability of the system.

There are various forms of FACTS devices: some are connected in series while others are connected in shunt or a combination of series and shunt like in our case with the UPFC. The UPFC base model and the UPFC injection model are presented in Figures 1 and 2.

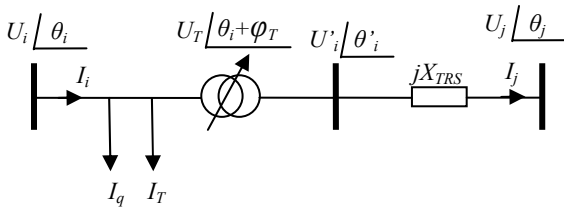


Fig.1. UPFC base model

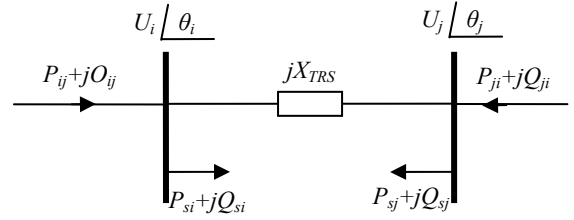


Fig.2. UPFC injection model

The equations associated with the UPFC are [7]:

$$p_{si} = \frac{U_i \cdot U_T}{X_{TRS}} \cdot \sin(\varphi_T) + U_i \cdot I_T \quad (18)$$

$$p_{sj} = -\frac{U_j \cdot U_T}{X_{TRS}} \cdot \sin(\theta_{ij} + \varphi_T) \quad (19)$$

$$Q_{si} = \frac{U_i \cdot U_T}{X_{TRS}} \cdot \cos(\varphi_T) + U_i \cdot I_q \quad (20)$$

$$Q_{sj} = -\frac{U_j \cdot U_T}{X_{TRS}} \cdot \cos(\theta_{ij} + \varphi_T) \quad (21)$$

With:

$$\theta_{ij} = \theta_i - \theta_j \quad (22)$$

We take in consideration the insertion of the UPFC between two buses  $i$  and  $j$  in the system model by adding to the diagonal element  $Y_{ii}$  and  $Y_{jj}$  of system admittance matrix without UPFC respectively the elements  $Y_{sii}$  and  $Y_{sij}$  given by:

$$Y_{sii} = \frac{(P_{si} - jQ_{si})}{|V_i|^2} \quad (21)$$

$$Y_{sij} = \frac{(P_{sj} - jQ_{sj})}{|V_j|^2} \quad (22)$$

$U_T$  magnitude of the injected voltage

$\varphi_T$  angle of the injected voltage

$I_q$  reactive parallel branch current

$I_T$  depends on the active power injected in the series branch

### 4. Energy Function of System with UPFC

In 1892, A. M. Lyapunov proposed his famous PHD dissertation the stability of equilibrium point of a nonlinear dynamic system “the general problem of stability of motion”. Many different Lyapunov functions have been tried since then, the first integral of motion, which is the sum of kinetic and potential energies seemed to have provided the best result [7]-[8]-[9].

In reference [9] we find a demonstration about how to construct an energy function for multi machines power system when machine are representing by the second order model without regulation.

Reference [10] gives an expression for an energy function for the  $i^{th}$  machine of system with detailed model based on the energy function given in [9].

The total energy of a multimachines power system is given by the following expressions:

$$V_T(t_0, t) = V_K(t_0, t) + V_P(t_0, t) \quad (23)$$

$$V_K(t_0, t) = 1/2 \cdot \sum_{i=1}^n M_i \cdot \omega_{i0}^2(t) \quad (24)$$

$$\begin{aligned} V_P(t_0, t) = & - \sum_{i=1}^n (P_{mi}(t) - (e_{di}^2(t) + e_{qi}^2(t)) G_{ii}) \cdot (\delta_{i0}(t) - \delta_{i0}(t_0)) \\ & + \sum_{i=1}^n \sum_{j=1, \neq i}^n H_{ij} \cdot \frac{(\delta_{i0}(t) - \delta_{i0}(t_0))(\cos \delta_{ij}(t_0) - \cos \delta_{ij}(t_e))}{(\delta_{i0}(t) - \delta_{j0}(t_0)) \cdot (\delta_{i0}(t) - \delta_{j0}(t_0))} \\ & + \sum_{i=1}^n \sum_{j=1, \neq i}^n F_{ij} \cdot \frac{(\delta_{i0}(t) - \delta_{i0}(t_0))(\sin \delta_{ij}(t_0) - \sin \delta_{ij}(t_e))}{(\delta_{i0}(t) - \delta_{j0}(t_0)) \cdot (\delta_{i0}(t) - \delta_{j0}(t_0))} \\ & + \sum_{i=1}^n \left( \frac{M_i}{M_T} (\delta_{i0}(t) - \delta_{i0}(t_0)) \sum_{j=1}^n (P_{mi} - P_{ej}(t_{0+})) \right) \quad (25) \end{aligned}$$

With:

$$H_{ij} = B_{ij} \cdot (e_{di} \cdot e_{dj}' + e_{qi} \cdot e_{qj}') - G_{ij} (e_{di} \cdot e_{qj}' + e_{qi} \cdot e_{dj}') \quad (26)$$

$$F_{ij} = G_{ij} \cdot (e_{di} \cdot e_{dj}' + e_{qi} \cdot e_{qj}') - B_{ij} (e_{di} \cdot e_{qj}' + e_{qi} \cdot e_{dj}') \quad (27)$$

For system with UPFC, we must add a term to represent the effect of UPFC and the transient energy function takes this expression:

$$V_T(t_0, t) = V_K(t_0, t) + V_P(t_0, t) + V_{UPFC} \quad (28)$$

In the case of UPFC, the energy function is equal to the total sum of the reactive powers [7]:

$$V_{UPFC} = Q_{si} + Q_{sj} \quad (29)$$

## 5. CCT Evaluation by PEBS Approach

In our study, the Structure Preserving Energy Function is considered. For this case, the CCT can be determined as the maximum value of an energy function along the system's fault-on trajectory. In this way, the evaluation of CCT for a particular fault is limited to just one fault-on numerical integration.

The flowchart presented in figure 3 describes a procedure to CCT evaluation in the case of SPEF and the use of PEBS method [11].

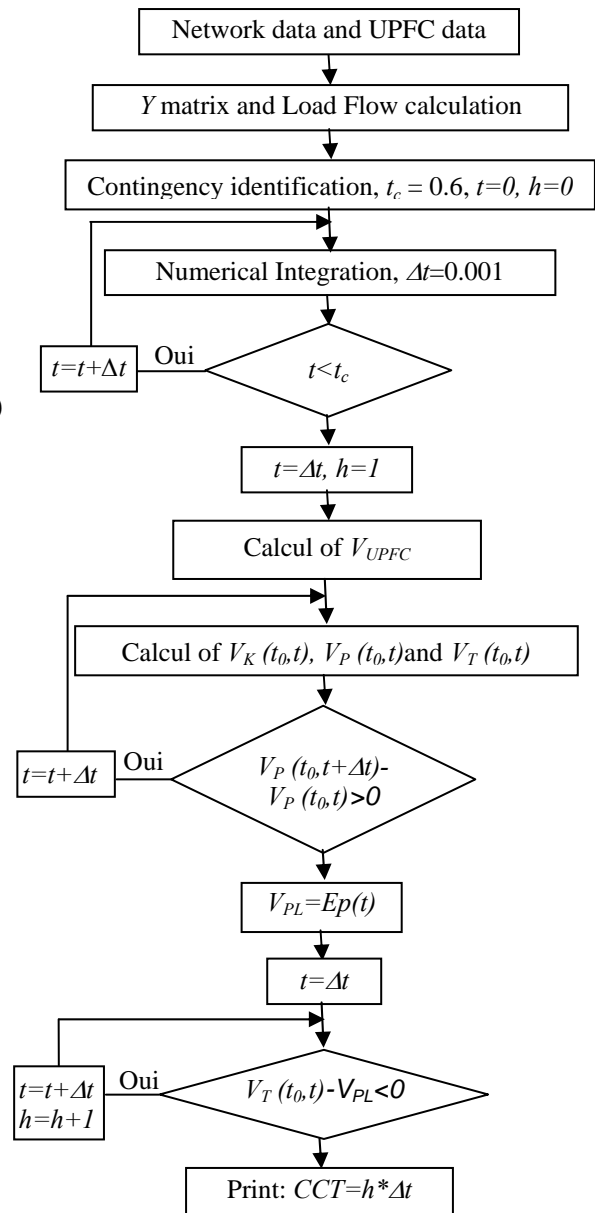


Fig.3. Flowchart of PEBS approach

- $t_c$  fault clearing time.
- $V_{PL}$  maximum value of the potential energy
- $\Delta t$  step integration

## 6. Simulation Results

The UPFC is connected between buses 2 and 10 (Figure 4) and its constant parameters considered for all simulations are:

$$U_T=0.03 \text{ pu}, I_q=0.03 \text{ pu}, \varphi_l=85^\circ \text{ and } X_{TRS}=0.06 \text{ pu.}$$

Data for speed regulation and excitation regulation systems are given in table 1 and 2 respectively.

The contingencies considered in this study are the buses three phase short circuit followed by opening the faulted line. The CCT evaluated by the numerical integration method and the proposed approach are given in Table 3.

TABLE 1. – Excitation Regulation System DATA

		1	2	3
$T_e$	$s$	0.100	0.100	0.100
$K_e$	-	50.00	50.00	50.00
$E_{xma}$	$pu$	+ 5.00	+ 5.00	+ 5.00
$E_{xmi}$	$pu$	- 5.00	- 5.00	- 5.00

TABLE 2. – Speed Regulation System DATA

Generator		1	2	3
$T_v$	$s$	5.0	5.0	5.0
$K_v$	-	0.5	0.5	0.5
$\sigma$	-	0.6	0.6	0.6
$P_{mma}$	$pu$	3.00	3.00	1.50
$P_{mni}$	$pu$	0.80	0.80	0.40

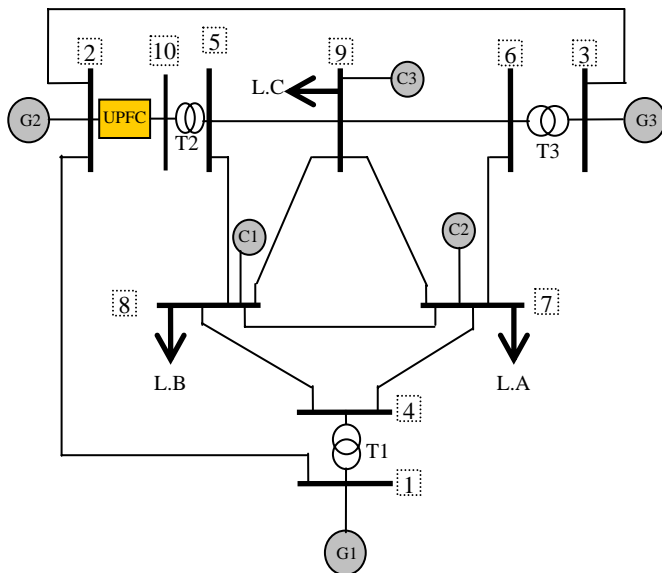


Fig.4. IEEE 3 machines 9 Bus test system

TABLE 3. - Critical Clearing Time (s)

Faulted Line	Faulted Bus	CCT of System			
		without UPFC		with UPFC	
		PEBS	NI	PEBS	NI
1-2	1	0.506	0.51	0.547	0.56
5-9	5	0.402	0.41	0.435	0.47
7-8	8	0.525	0.54	0.539	0.55
8-9	9	0.547	0.56	0.549	0.58
4-7	4	0.510	0.52	0.525	0.55

Results in table 3 demonstrate the ability of UPFC to enhance power system transient stability. They demonstrate also that the proposed algorithm based on PEBS can evaluate the CCT but they give conservative values. Nevertheless, they can be used for on-line dynamic security assessment because they are faster than numerical integration approach.

## 7. Conclusion

In this study, a new computationally fast tool is developed to evaluate the system transient stability Critical Clearing Time in a range of a hundred of milliseconds. The PEBS approach is used to achieve this goal. The used Transient Energy Function considers a detailed model of generators and containing an additional term which represent the UPFC effect. Results show the ability of the proposed algorithm to evaluate the first swing transient stability of power system with UPFC. A generalised software MATLAB tool is elaborated for both numerical integration and energy function methods. Simulations performed through the studied systems give promising results. For further work, we propose to use other approaches based on Energy Function to precisely evaluate the CCT and to use this tool for assessing system transient stability margins, such that proper emergency control mechanisms can be activated in order to maintain stability and avoid large catastrophic outages.

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